Yasuto Mushiake, "Constant-Impedance Antennas," J. IECE Japan, Vol.48, No.4, April 1965, pp. 580-584.

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1. Introduction

Constant-impedance antennas are antennas whose input impedance does not change with respect to the frequency. There are two types of the constant-impedance antennas: first type antennas have frequency-independent radiation characteristics as well as constant-impedance property and second type antennas have radiation characteristics which change with respect to the frequency. We can obtain constant-impedance antennas theoretically, but actually we cannot obtain the perfect constant-impedance. However, we can realize antennas whose input impedance is almost constant over very broadband. Therefore, the constant-impedance antennas are very interesting from the points of view of the application of antennas as well as the antenna theory.

It is well known that the self-similar antennas such as the biconical antenna with infinite length are the constant-impedance antennas. Another type of the constant-impedance antennas having different shapes has been shown by the present authors in 1948^{(1)~(3)}. Rumsey has proposed and developed frequency independent antennas, whose radiation characteristics as well as input impedance are independent of frequency, in order to apply these constant-impedance antennas to broadband antennas⁽⁴⁾. However, the condition of the constant radiation characteristics is difficult to realize and the definition of such condition is limited. The constant-impedance antennas described in the present paper involve antennas in a broad sense.

In this paper, classification of the constant-impedance antennas is described. Then, our expertise on the constant-impedance antennas is presented based on our recent works.

2. Classification of the constant-impedance antennas

The constant-impedance antennas developed before can be divided into the following manner from the points of view of their principles and shapes.

- (i) Self-complementary antennas
 - (a) Two-terminal planar self-complementary antennas (rotationally symmetry type and line symmetry type)
 - (b) Multi-terminal planar self-complementary antennas (rotationally symmetry)
 - (c) Three-dimensional self-complementary antennas (line symmetry)
- (ii) Self-similar antennas
 - (a) Biconical type self-similar antennas

- (b) Spiral type self-similar antennas
- (c) Rolled triangular-sheet type self-similar antennas

In these antennas, self-complementary antennas were proposed by the present authors for the first time. Since two-terminal planar self-complementary antennas were already reported⁽²⁾, other self-complementary antennas are described below.

3. Multi-terminal planar self-complementary antennas

The extension of two-terminal planar self-complementary antennas to multi-terminal planar selfcomplementary antennas have been carried out by Deschamps⁽⁴⁾ and the author⁽⁶⁾ independently. The outline of the research is described below focusing on our research works.

Let us consider *n*-terminal self-complementary antennas shown in Fig. 1. General relations hold as follows,

$$V_s = I_s'/2, \ I_s = 2 \ V_s'/Z_0^2$$
 (1)

where $Z_0 \approx 120\pi \approx 376.7[\Omega]$ is the intrinsic impedance of the space. The relation between the ratios of the voltage and the current of two antennas Z_s , Z'_s are expressed as

$$Z_{s} = \frac{V_{s}}{I_{s}} = \frac{Z_{0}^{2}}{4} \frac{I_{s'}}{V_{s'}} = \frac{Z_{0}^{2}}{4} \frac{1}{Z_{s'}}$$
(2)

When the excitation of the ring connection or the star connection is symmetrical *n*-phase electric sources with *m*th-order rotation, V_s , I_s , V'_s , I'_s are symmetrical multi-phase sources because of their structures and eq. (1). If the structures in Fig. 1 are self-complementary structures, the antennas (a) and (b) are the same antennas except for their excitations, i.e., the ring connection and the star connection. In general, the relation between the impedances of the ring connection and that of the star connection holds

$$Z_{mn} = 4 Z'_{mn} \sin^2(m \pi/n)$$
(3)

$$Z_{mn}, Z'_{mn} \text{ correspond to } Z_s, Z'_s \text{ in eq. (2), respectively, and by using eqs. (2), (3), they are given by (3)$$

$$Z_{mn} = Z_0 \sin(m \pi/n), \ Z_{mn'} = Z_0 / \{4 \sin(m \pi/n)\}$$
(4)

where $m = 1, 2, \dots, n-1$. Since this equation holds independently of frequency, it is found that the multi-terminal planar self-complementary antennas are constant-impedance antennas.

The discussion described above is about the case of symmetrical excitation, however, in the case of arbitrary excitation, the excitation can be expressed by a linear combination of *m* symmetrical excitations and the impedance is constant for each excitation. As a result, constant-impedance antennas having various value of impedance can be obtained, where the mutual impedance between terminals is also constant. The impedances of planar 4-terminal self-complementary antennas are shown in Table 1. For example, when the terminals 3 and 4 are shorted, the input impedance of

terminals 1 and 2 is $120\sqrt{2\pi}/(1+\sqrt{2})[\Omega]$.



Fig. 1 n-terminal self-complementary antennas excited by n-phase electric sources.

Table 1	Examples	of self impedat	nce and mutua	l impedance	of planar 4	-terminal s	self-
	complem	entary antennas	5.				

		and the second	
Connection	Self impedance	Mutual impedance	
	$60 \sqrt{2}\pi,$ [Ohms]	0	
• <u> </u>	$(\sqrt{2}+1)30\pi$ [Ohms]	$(\sqrt{2}-1)30 \pi$ [Ohms]	
	$\left(\sqrt{2}+1\right)15\sqrt{2}\pi$ [Ohms]	$(\sqrt{2} - 1)15 \sqrt{2}\pi$ [Ohms]	

4. Three-dimensional self-complementary antennas

Extension of the concept of self-complementary antenna to three-dimensional plates was performed by the present authors⁽⁸⁾. Let us consider a geometry composed of n conducting half-

sheets which are axially symmetric, as shown in Fig. 2. These plates are divided into symmetric planes that are complementary each other and having the same shapes. Then, the plates are divided into head and tail surfaces as well as right and left surfaces as shown in Fig. 3(a), and we obtain complementary antennas in 2n wedge spaces with an angle of π/n as shown in Fig. 3(b). The voltages and the currents in Figs. 3(a) and Fig. 3(b) are given by

$$V = V_{1} = V_{2} = \dots = V_{n} = V_{0}$$

$$I = 2 n I_{0}, I_{1} = I_{2} = \dots = I_{n} = I_{0}$$
(5)

$$V' = V_{1}' = V_{2}' = \dots = V_{n}' = V_{0}'$$

$$I' = 2 n I_{0}', I_{1}' = I_{2}' = \dots = I_{n} = I_{0}$$
(6)

Since the geometry and the feeding are symmetric, it can be shown that the following equation holds in the wedge spaces with the angle of π/n , in the same way as two-dimensional planar structure.

$$V_0 = I_0', \quad I_0 = V_0' / Z_0^2$$
 (7)

and we obtain

$$Z = \frac{V}{I} = \frac{V_0}{2 n I_0} = \frac{Z_0^2 I_0'}{2 n V_0'} = \frac{Z_0^2}{4 n^2}$$

$$\cdot \frac{I'}{V'} = \left(\frac{Z_0}{2 n}\right)^2 \frac{1}{Z'}$$
(8)

Since the antennas shown in Figs. 3(a) and Fig. 3(b) are exactly the same, these antennas are selfcomplementary antennas and Z = Z'. Therefore, Z is given by

$$Z = Z_0/2 \, n \simeq 60 \, \pi/n \simeq 188/n \tag{9}$$

In eq. (9), n=1 corresponds to original planar self-complementary antenna. Measured input resistance of antenna of n=2 composed of orthogonal two aluminum plates with side lengths of 1 m x 1 m is shown in Fig. 4.



Fig. 2 Three-dimensional self-complementary antennas (n=2).



Fig. 3 Decomposition of three –dimensional self-complementary antenna (n=2).



Fig. 4 Measured input resistance of three-dimensional self-complementary antenna (n=2).

5. Self-similar antennas

Self-similar object is defined by the fact that the structure is similar to a part of itself. Antennas having self-similar structures are called 'self-similar antennas'. The self-similar antennas are constant-impedance antennas because of the similarity theorem of electromagnetic field. There are two types of antennas: one type antennas are the antennas such as the biconical antennas whose linear dimensions reduce by a given factor, then they have the similar shape, and other type antennas are antennas such as the spiral antennas in which the dimensions reduce and antennas rotate, then they have the similar shape. Rolled triangular-sheet antenna, which has been proposed by the present authors, is one of the spiral type antennas. Many works on these antennas were reported^{(4),(11),(12)} and we omitted here about the self-similar antennas.

6. Impedance of log-periodic antennas

Log-periodic antennas are considered to be related to constant-impedance antennas. Log-

periodic antennas are defined such that when the dimensions increases by a factor of K^n (*K*: constant, *n*: integer number), they have the same shape as the original antennas. Therefore, the characteristics vary in a log-periodically manner as a function of frequency because of the similarity theorem and there is no guarantee to obtain a constant-impedance property. There are misunderstandings that the log-periodic antennas are always broadband antennas. These misunderstandings are considered to be due to the fact that so-called log-periodic antenna⁽¹³⁾ developed by Du Harmel et al. have broadband characteristics, because log-periodic antenna is a modification of self-complementary antenna. The author has already reported about this misunderstandings, and recently we performed experimental investigation of the input impedance of log-periodic antennas. The results are shown in Figs 5, 6 and 7. It is noted that the log-periodic modified self-complementary antenna shown in Fig. 5 has broadband characteristics as shown in

Fig. 5. On the other hand, input impedance of another antenna, where one wing of two half structure of antennas shown in Fig. 5 is upside-down, varies significantly with frequency. We would like to stress that log-periodic antennas are not always the broadband property.



Fig. 5 Measured input impedance of log-periodic modified self-complementary antenna.



Fig. 6 Measured input impedance of log-periodic antenna arranged in anti-complementary manner.



Fig. 7 Comparison of measured input resistance of log-periodic modified self-complementary antenna and log-periodic anti-complementary antenna.

7. Constant-impedance antennas and broadband antennas

Constant-impedance is one of the required property of broadband antennas. We would like to describe a few problems to realize the constant-impedance antennas. The constant-impedance antennas theoretically require infinite structures, but practical antennas are finite structures. Usually, the truncation effect is not so significant, and almost constant-impedance can be obtained. The maximum size of the antennas gives the lowest limit of frequency and the size of feeding structure gives the highest limit of frequency. The constant radiation property is more difficult to realize than the constant impedance. The condition of constant radiation property is that the current decreases rapidly as the distance from the driving point increases⁽¹⁵⁾. This condition can also reduce the truncation effect.

8. Conclusion

We presented our expertise on the constant-impedance antennas based on our recent works of self-complementary antennas. We hope that the theoretical and application researches on these antennas will be expanded and the results become useful for the design of broadband antennas.

References

(1) Y. Mushiake, "The Input Impedance of a Slit Antenna," Joint Convention Record of Tohoku

Sections of IEE and IECE of Japan, pp. 25-26, June 1948.

- (2) Y. Mushiake, "The input impedances of slit antennas," J. IEE Japan, Vol.69, No.3, pp. 87-88, March 1949.
- (3) S. Uda and Y. Mushiake, "The Input Impedances of Slit Antennas," Technical Report of Tohoku, University, Vol.14, No.1, pp. 46-59. Sep. 1949.
- (4) V. H. Rumsey, "Frequency independent antennas," 1957 IRE National Conv. Record, pt. I, pp.114-118, March 1957.
- (5) G. A. Deschamps, "Impedance properties of complementary multiterminal planar structures. IEEE Trans. Antennas Propagat. Vol.AP-7, No. 5, pp. 371-379, Dec. 1959.
- (6) Y. Mushiake, "Multi-terminal constant impedance antenna," 1959 National Convention Record of IECE of Japan, p. 89, Oct. 1959.
- (7) For example, Yasuto Mushiake, *Antennas, and Radio Propagation*, Corona Publishing, Tokyo, Japan, 1961.
- (8) Y. Mushiake and H. Saito, "Three-dimensional self-complementary antenna," Joint Convention Record of Four Japanese Institutes Related to Electrical Engineering, Pt. 15, No. 1212, April 1963.
- (9) J. R. MacDougal, H. Mohri, S. Adachi, and Y. Mushiake, "Rolled triangular-sheet antennas," Joint Convention Record of Four Japanese Institutes Related to Electrical Engineering, April 1957.
- (10) Y. Mushiake, "Rolled triangular-sheet antenna as a self-complementary antenna," Joint Convention Record of Four Japanese Institutes Related to Electrical Engineering, July 1960.
- (11)R. L. Carrel, "The characteristic impedance of two infinite cones of arbitrary cross section," IRE Trans. Antennas Propagat., Vol. AP-8, No.2, pp. 197-201, April 1958.
- (12) J. D. Dyson, "The equiangular spiral antenna," IRE Trans. Antennas Propagat., Vol.AP-7, No.2, pp. 181-187, April 1959.
- (13)R. H. DuHamel and D. E. Isbell, "Broadband logarithmically periodic antenna structures. 1957 IRE National Conv. Record, pt. I, pp.119-128, March 1957.
- (14) Y. Mushiake, "Principle of Log-Periodic Antenna, (comments)," Broadcast Engineering, Vol.13, No. 8, pp. 441-444, August 1960.
- (15)E. C. Jordan, G. A. Deschamps, J. D. Dyson, and R. E. Mayes, "Developments in broadband antennas," IEEE Spectrum, Vol.1, No.4, pp. 58-71, April 1964.