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Papers

An Analysis on Single Wavelength Oscillation of Semiconductor Laser at High Speed Pulse Modulation

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SUMMARY In this paper, an analysis was made on the variation of oscillating mode numbers of a semiconductor laser at high speed pulse modulation by solving multi-mode rate equations which relate a quasi-Fermi level ζ to photon numbers. The laser gain was assumed to be homogeneous and has a dependency of $\sqrt{\Delta}(\zeta-\Delta)$, where Δ is $\hbar\omega - E_g$ (relative photon energy). Single mode oscillation conditions for short pulse modulation were obtained associated with following two cases. A) One is the criterion how wide the mode separation must be made as of a short cavity laser and of an integrated twin-guide (ITG) laser. B) The other is the condition how large we must give an excess loss to the unwanted modes such as of the ITG laser, of a distributed feedback (DFB) laser, and of a distributed Bragg reflector (DBR) laser.

1. Introduction

Studies in which we aim at stabilizing the modulation characteristics of a semiconductor laser have been made with great efforts. It was shown that the relaxation oscillation can be reduced by injecting light from another laser or by the use of self-injection[1] and by connecting a resonance circuit[2]. Recently it was found that the adjustment of stripe width to an optimum value suppresses the relaxation oscillation because the diffusion of carriers works well[3].

On the other hand, reducing the spectral width of a laser is one of the problems to be solved, when the laser is used as a light source in optical fiber communication because the fiber has a material dispersion of a considerable amount. When the light pulse is transmitted through a glass fiber which has a material dispersion nearly of 10 ps/km·Å in 0.85 μm region of wavelength[4], its width exceeds 3 ns if the fiber length is 10 km and the spectral width 30 Å as in a usual multimode semiconductor laser. If it is necessary for us to limit the pulse width less than 0.5 ns after 10 km transmission, the spectral width of the laser must be smaller than 5 Å, even at high-speed pulse modulation.

By thist ime, the observations of transient variation in oscillating mode numbers were tried[5]~[9]. The decreasing tendency of spectral width versus the time after the onset of light pulse was theoretically discussed[7]. These works concern mainly with transient variation of spectra when lasers are modulated with square shape pulses. The authors pointed out that it is necessary to know how mode numbers vary when the semiconductor laser is modulated with a short current pulse and to obtain single mode oscillation conditions for some types of semiconductor lasers[10].

In this paper, the variation of oscillating mode numbers is investigated by solving the multimode rate equations, in the case that the laser is modulated with a high speed squared-cosine pulse. Single wavelength oscillation conditions are related to A) a mode separation of a short cavity laser[11] and of an integrated twin-guide (ITG) laser[12], and to B) a necessary loss for unwanted modes as in the ITG laser, in a distributed feedback (DFB) laser[13], and in a distributed Bragg reflector (DBR) laser[14].

2. Rate Equations

Let us use in this analysis the multi-mode rate equations expressed by the following form,

$$\frac{dS_i}{dt} = -\frac{1}{\tau_{pi}} S_i + G(\Delta_i, \zeta) S_i + \frac{C_i}{\tau_s} N(\zeta) \tag{1}$$

$$\frac{dN(\zeta)}{dt} = \frac{J}{ed} - \frac{1}{\tau_s} N(\zeta) - \sum_i G(\Delta_i, \zeta) S_i \tag{2}$$

where

S_i	photon density of mode i
$N(\zeta)$	injected carrier density into the conduction band as a function of ζ
ζ	quasi-Fermi level
$\hbar\omega_i$	photon energy of mode i
E_g	band gap energy
Δ_i	$\hbar\omega_i - E_g$
$G(\Delta_i, \zeta)$	gain as a function of ζ and Δ_i
τ_{pi}	photon life time of mode i
τ_s	carrier life time
e	electron charge
d	thickness of an active layer
C_i	spontaneous emission factor[15,16]
J	injection current.

The above equations have been extended in order to express multi-mode operations from quantum mechanical rate equations[17]. The spontaneous term is written

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in the usual form[15,16], that is $N(\zeta)/\tau_s$. In the last term of Eq. (1) we use the spontaneous emission factor C_i which was introduced by Boers et al[15] and suematsu et al[16]. These are small modifications from equations in Ref. (17).

It is assumed that minority carriers injected into the conduction band are brought into quasi-eqilibrium in a time perod of the order of 1 ps which is much shorter than the time period of the variation of optical field. A quasi-Fermi level ζ is used as a dynamic parameter on behalf of a carrier number N in order to express the variation of the gain for each mode separately. We expand $N(\zeta)$ in terms of a quasi-Fermi level increment $\zeta-\zeta_{th}$ in the same manner stated in Ref. (17) as follows;

$$N(\zeta)=N(\zeta_{th})+\left.\frac{dN}{d\zeta}\right|_{\zeta_{th}}(\zeta-\zeta_{th}) \tag{3}$$

where ζ_{th} is threshold quasi-Fermi level at stationary condition. We put $N(\zeta_{th})=N_{th}$ for simplicity, hereafter. In this paper, the numerical values are associated with pure GaAs[17].

The gain factor $G(\Delta_i,\zeta)$ in this analysis is based on the assumption that the transition is from band to band and spatially homogeneous. The wavelength dependency of the gain, therefore, can be given by the product of $\sqrt{\Delta_i}$ and the quasi-Fermi level increment $\zeta-\Delta_i$ as introduced in Ref. (17);

$$G(\Delta_i,\zeta)=G_{th}(3^{3/2}/2)\sqrt{\Delta_i/\zeta_{th}}(\zeta/\zeta_{th}-\Delta_i/\zeta_{th}) \tag{4}$$

The gain takes its maximum at $\Delta_i=\zeta/3$. We assume that oscillation occurs at $\Delta_i=\zeta_{th}/3$ at stationary condition, when the quasi-Fermi level reaches the threshold value ζ_{th}. The gain saturates and takes the value G_{th}.

Next, we consider that the wavelength dependency and shift of a transient gain are also expressed by Eq. (4). We allow an overswing of the quasi-Fermi level ζ as shown in Fig. 1. The vertical axis expresses the normalized gain G/G_{th}. We see from Eq. (4) that the full width of photon energy where the laser has positive gain is equal to ζ_{th}. The width of wavelength $\Delta\lambda_G$ associated with ζ_{th} is given by the equation

$$\Delta\lambda_G=(12398/E_g^2)\zeta_{th} \quad (\text{Å}) \tag{5}$$

where E_g and ζ_{th} are given in eV.
In Ref. (17), the threshold Fermi level ζ_{th} is calculated by using definite values of parameters associated with pure GaAs. As it may depend on the cavity loss, temperature, the aluminum content in the active region, doping level and so on, we take $\Delta\lambda_G$ as a parameter.

The longitudinal mode separation is denoted by $\Delta\lambda$. As shown in Fig. 1, the mode 1 is set at the maximum of threshold gain as mentioned above. The mode 2 and 3 are set at shorter wavelength side, and the mode 2' at longer wavelength side, respectively.

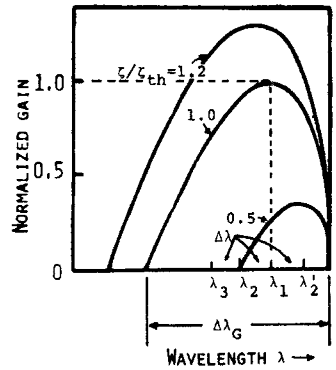


Fig. 1 Normalized gain G/G_{th} vs wavelength for normalized quasi-Fermi level ζ/ζ_{th}.

3. Multi-mode Oscillation in Pulse Modulation

Because the gain is assumed to be homogeneous, the rate equations lead to single mode oscillation in the stationary condition. First, the rate equations were solved by putting d/dt=0. Solid lines in Fig. 2 show photon numbers of mode 1, 2, and 3 versus DC current J normalized by the threshold value J_{th}. Here we have assumed that spontaneous emission factor $C_i=10^{-6}$. As C_i increases photons of each mode increase below the threshold, but over the threshold the mode 2 and 3 reach their satulated values.

On the other hand the broken lines in Fig. 2 show the variation of photons versus the input current for a high speed pulse modulation. An input current shape is

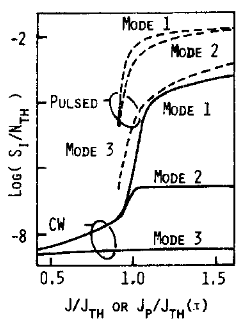


Fig. 2 Photon numbers of mode 1, 2, and 3, for cw and pulsed oscillation.

assumed to be a squared cosine as given by the following equation

$$\begin{aligned} J &= (J_p/2)\{1-\cos(\pi t/\tau)\} && (0<t<2\tau) \\ &= 0 && (t<0,\ 2\tau<t) \end{aligned} \tag{6}$$

where J_p is a peak value and τ is a half width. The parameter $J_{th}(\tau)$ in Fig. 2 is threshold current at pulse modulation and expressed by the equation[18]

$$J_{th}(\tau)=J_{th}^{cw}(1+\tau_s/\tau) \tag{7}$$

where J_{th}^{cw} is threshold current at cw condition. In Fig. 2 we have assumed that $C_i=10^{-6}$, $\tau_s/\tau_p=10^3$, $\tau/\tau_s=0.5$, and $\Delta\lambda/\Delta\lambda_G=7\%$. From this result, we see that the intensity of mode 2 and 3 as well as mode 1 increase above the threshold. It was confirmed that the quasi-Fermi level exceeds its threshold value transiently. It was also made clear that the shorter wavelength modes 2 and 3 tend to oscillate at high pumping level and higher speed modulation, but the longer wavelength mode 2' is not in excess of mode 3.

4. Single-wavelength Oscillation Conditions

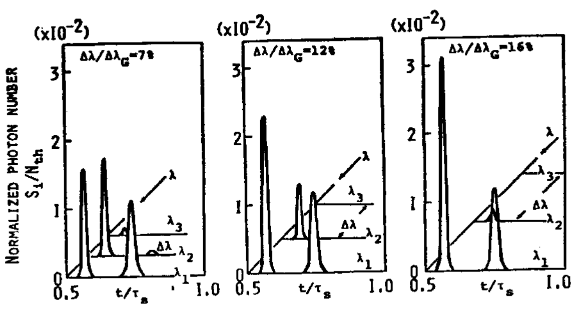
In section 3, it was seen how multi-mode oscillation occurs transiently in a laser which is modulated by a short pulse current. It may be able to obtain single mode criteria if we solve rate equations in case that we try to suppress the multi-mode oscillation by reducing gains or increasing losses of unwanted modes i. e. mode 2, 3, and 2'.

Since it has been shown in Section 3 that longer wavelength modes do not exceed shorter wavelength modes, we are interested in suppressing shorter wavelength modes. Hereafter, C_i is assumed to be 10^{-6}. This value does not take an important role. Let us condiser single wavelength oscillation conditions for two cases.

4-1 Criterion as a function of mode separation

In the first case, we consider widening a mode separation which is often tried to reduce mode numbers in various lasers. By this scheme, we can expect that the gain for mode 2 and 3 will become smaller even in pulse modulation and can not exceed the loss. This consideration is useful for deciding the cavity length of a short cavity laser[11], for deciding the separation between modes which have lower loss than neighboring Fabry-Perot modes in integrated twin-guide lasers[12,13], and examining the separation of $m=0$ and $m=1$ mode in distributed feedback lasers[13].

Fig. 3 shows normalized photon numbers for the mode 1, 2, and 3 when the normalized mode separation



(a) (b) (c)

Fig. 3 Solutions of rate equations for various values of relative mode separation $\Delta\lambda/\Delta\lambda_G$. $\tau/\tau_s=0.5$, $\tau_s/\tau_p=10^3$, and $J_p/J_{th}(\tau)=1.5$.

$\Delta\lambda/\Delta\lambda_G$ is varied. One of the horizontal axes expresses time normalized by a carrier life time τ_s and the other does wavelength. Here we assume that $\tau/\tau_s=0.5$, $\tau_s/\tau_p=10^3$, and $J_p/J_{th}(\tau)=1.5$. Since the carrier life time τ_s of GaAs DH lasers is nearly equal to 3 ns around room temperatures[20], the first assumption corresponds to $\tau=1.5$ ns. Fig. 3 (a) is with $\Delta\lambda/\Delta\lambda_G=7\%$. If we assume $\Delta\lambda_G=50$ Å, the mode separation $\Delta\lambda$ corresponds to 3.5 Å which is the value associated with usual Fabry-Perot lasers. Shorter wavelength mode 2 and 3 also oscillate. On the other hand, Fig. 3 (b) and (c) are associated with $\Delta\lambda/\Delta\lambda_G=12\%$ and 16%, respectively. From these results, it is seen that photons of unwanted shorter wavelength modes decrease as the mode separation increases.

By examining the first large peaks of mode 1 and mode 2, the condition for single wavelength oscillation is estimated. Fig. 4 shows how the mode 2 decreases

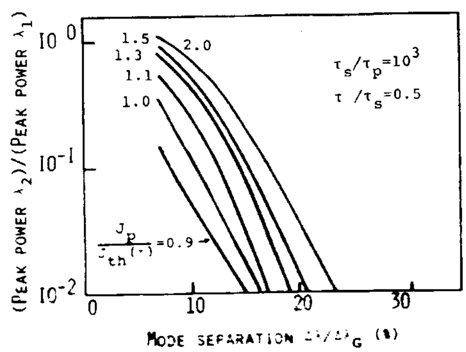


Fig. 4 The ratio of thepeak powers of mode 2 and mode 1 vs the relative mode separation $\Delta\lambda/\Delta\lambda_G$.

in comparison with mode 1 when $\Delta\lambda/\Delta\lambda_G$ is changed. For higher speed modulation, that is, $\tau/\tau_s=0.5$, the necessary mode separation for single mode oscillation must be larger than the case $\tau/\tau_s=1.0$.

If we define the single wavelength oscillation condition as (the peak power of mode 2)/(the peak power of mode 1) = 0.01, necessary conditions for some values of τ/τ_s and τ_s/τ_p are obtained versus $J_p/J_{th}(\tau)$ and the result is shown in Fig. 5. From this figure, we see that the necessary mode separation must be larger than

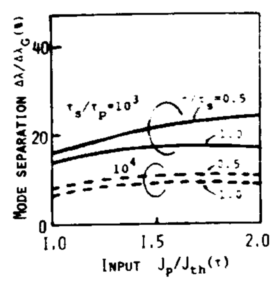


Fig. 5 Necessary mode separation in order to achieve single mode oscillation vs the normalized input current $J_p/J_{th}(\tau)$.

24 %, when $\tau/\tau_s=0.5$, $\tau_s/\tau_{p_0}=10^3$, and $J_p/J_{th}(\tau)=2.0$. If we take $\Delta\lambda_G=50\,\text{Å}$, $\Delta\lambda=12\,\text{Å}$. The cavity length L of a Fabry-Perot laser is given by the equation

$$L=\lambda^2/(2n\Delta\lambda) \tag{8}$$

where n is effective refractive index of an active layer. By substituting $\Delta\lambda=12\,\text{Å}$, $\lambda=8500\,\text{Å}$, and $n=3.5$, we get $L=86\,\mu$m.

4-2 Criterion as a function of loss for unwanted modes

In the second case, we consider the case that some losses are given to unwanted modes in order to achieve single wavelength oscillation. This is related to the special cavity construction which is made in order to suppress neighboring Fabry-Perot modes such as in the integrated twin-guide laser[12,19], in a distributed Bragg reflector laser[14], and in the distributed feedback laser[13]. Since the loss for photons in a laser cavity can be related to the inverse of the photon lifetime τ_p, the additional loss to unwanted modes are represented by the reduction of τ_p. This photon lifetime difference is denoted by $\Delta\tau_p$ and a relative loss $\Delta\tau_p/\tau_{p1}$ is taken as a parameter, where τ_{p1} is the lifetime of mode 1. The relation between the gain and loss is schematically shown in Fig. 6.

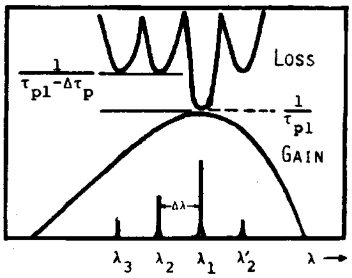


Fig. 6 The relation between the gain and excess loss given to unwanted modes.

Some solutions of the rate equations are shown in Fig. 7, where we have assumed that $\tau/\tau_s=0.5$, $\tau_s/\tau_p=10^3$, $J_p/J_{th}(\tau)=1.5$, and $\Delta\lambda/\Delta\lambda_G=7\%$. The ratio of the first peaks associated with mode 2 and mode 1 is given versus the excess loss $\Delta\tau_p/\tau_{p1}$ for some excitation levels and is shown in Fig. 8. The additional

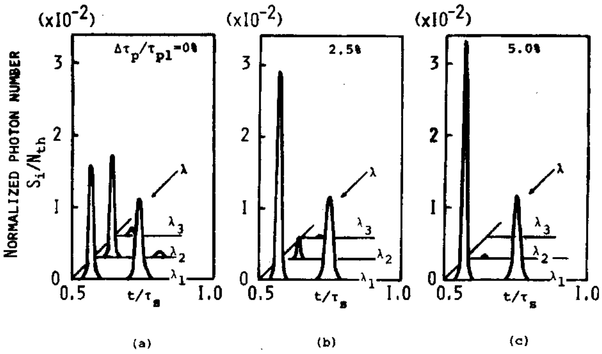


Fig. 7 Solutions of rate equations for various values of relative excess loss given to unwanted modes $\Delta\tau_p/\tau_{p1}$, $\Delta\lambda/\Delta\lambda_G=7$, $\tau/\tau_s=0.5$, $\tau_s/\tau_p=10^3$, and $J_p/J_{th}(\tau)=1.5$.

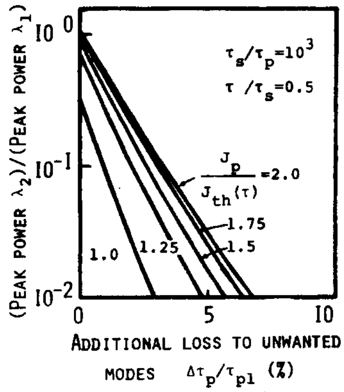


Fig. 8 The ratio of the first peaks of mode 2 and mode 1 vs the relative value of excess loss given to unwanted modes. $\Delta\lambda/\Delta\lambda_G=7\%$.

loss to unwanted modes that is necessary to keep (the peak power of mode 2) / (the peak power of mode 1) = 0.01 is plotted in Fig. 9 versus $J_p/J_{th}(\tau)$. The relative mode separation $\Delta\lambda/\Delta\lambda_G$ has been fixed to 7 %.

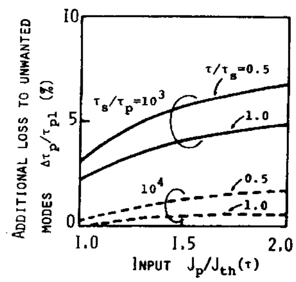


Fig. 9 Necessary additional loss to unwanted modes $\Delta\tau_p/\tau_{p1}$ vs the normalized input current $J_p/J_{th}(\tau)$. $\Delta\lambda/\Delta\lambda_G=7\%$.

In practical integrated twin-guide lasers[19] DFB lasers[21], and DBR lasers, the loss difference between the center mode and neighboring Fabry-Perot modes is

Table 1.

τ_s/τ_p	τ/τ_s	$\Delta\lambda/\Delta\lambda_G$ (%) (Short cavity, ITG,DFB)	Cavity length* $L(\mu m)\times\Delta\lambda_G$(Å)	$\Delta\tau_p/\tau_{p1}$# (%) (ITG,DFB,DBR)
10^3	0.5	24	4.3×10^3	7
	1.0	19	5.4×10^3	5
	2.0	16	6.5×10^3	4
10^4	0.5	12	8.6×10^3	2
	1.0	10	1.0×10^4	1
	2.0	8	1.3×10^4	1

#) $\Delta\lambda/\Delta\lambda_G$ = 7 %. *) λ = 8500 Å, n = 3.5.

10~20% at least. It can be said, therefore, that the single mode condition holds for these lasers also at high speed modulation.

Some numerical results which give the single wavelength oscillation criteria in the schemes stated in this paper are tabulated in Table I.

5. Conclusion

In this paper, multi-mode behaviors of semiconductor lasers at high speed pulse modulation are theoretically investigated by solving the multi-mode rate equations. The gain is assumed to be homogeneous and the excitation is given by a squared-cosine pulse current.

From this analysis it has been made clear how homogeneous semiconductor laser oscillates in multimode when it is modulated by a short pulse. It has been shown that shorter wavelength modes tend to strongly oscillate compared with longer wavelength modes due to asymmetric variation of the transient gain.

This single wavelength criteria are obtained for two cases as follows: A) Widening the mode separation of lasers, and B) Giving the losses to unwanted modes. The results and consideration given in this paper will be applicable to the cavity design of lasers which aim at single mode oscillation, when they are modulated by short pulses.

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