Instantaneous Reactive Power Compensators Comprising Switching Devices without Energy Storage Components

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Abstract—**The conventional reactive power in single-phase or three-phase circuits has been defined on the basis of the average value concept for sinusoidal voltage and current waveforms in steady states. The instantaneous reactive power in three-phase circuits is defined on the basis of the instantaneous value concept for arbitrary voltage and current waveforms, including transient states. A new instantaneous reactive power compensator comprising switching devices is proposed which requires practically no energy storage components.**

INTRODUCTION

VARIOUS TYPES of reactive power compensators has been researched and developed to provide power factor correction [1]-[5]. Notably, the static reactive power compensator comprising switching devices, which requires practically no energy storage components such as capacitors or reactors, was proposed by Gyugyi and others [6]-[8]. However, it has been considered that the compensator can eliminate only the fundamental reactive power in steady states. The generalized control strategy including the compensation of the fundamental reactive power in transient states and the harmonic currents has not yet been discussed in detail.

In this paper, the instantaneous imaginary power, which is a new electrical quantity, is introduced in three-phase circuits. Then, the instantaneous reactive power is defined as a unique value for arbitrary three-phase voltage and current waveforms including all distorted waveforms, by using the abovementioned instantaneous imaginary power. According to the theory developed in this paper, a new instantaneous reactive power compensator is proposed, comprising switching devices without energy storage components. This compensator can eliminate not only the fundamental reactive power in transient states but also some harmonic currents. For example, the harmonic currents having the frequencies of $f \pm 6f_o$ in a three-phase-to-three-phase naturally commutated cycloconverter can be eliminated, where f is the input frequency and f_o is the output frequency. The validity of the compensator is confirmed by experiments.

INSTANTANEOUS REACTIVE POWER THEORY

Definition of Instantaneous Imaginary Power

To deal with instantaneous voltages and currents in three-phase circuits mathematically, it is adequate to express their quantities as the instantaneous space vectors. For simplicity, the three-phase voltages and currents excluding zero-phase sequence components will be considered in the following.

In *a-b-c* coordinates, the *a*, *b*, and *c* axes are fixed on the same plane, apart from each other by $2\pi/3$, as shown in Fig. 1. The instantaneous space vectors, e_a and i_a are set on the a axis, and their amplitude and (+, –) direction vary with the passage of time. In the same way, e_b and i_b are on the b axis, and e_c and i_c are on the c axis. These space vectors are easily transformed into α-β coordinates as follows:

$$\begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} = \sqrt{\tfrac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\tfrac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{2}$$

where the α and β axes are the orthogonal coordinates. Necessarily, e_α and i_α are on the α axis, and e_β and i_β are on the β axis. Their amplitude and (+, –) direction vary with the passage of time.

Fig. 2 shows the instantaneous space vectors on the α-β coordinates. The conventional instantaneous power on the three-phase circuit can be defined as follows:

$$p = e_\alpha \cdot i_\alpha + e_\beta \cdot i_\beta \tag{3}$$

where p is equal to the conventional equation:

$$p = e_a i_a + e_b i_b + e_c i_c.$$

In order to define the instantaneous reactive power, the authors introduce the instantaneous imaginary power space vector defined by

$$q = e_\alpha \times i_\beta + e_\beta \times i_\alpha. \tag{4}$$

As shown in Fig. 2, this space vector is the imaginary axis vector and is perpendicular to the real plane on the α–β coordinates, to be in compliance with the right-hand rule. Taking into consideration that e_α is parallel to i_α and e_β to i_β, and that e_α is perpendicular to i_β and e_β to i_α, the conventional instantaneous power, p and the instantaneous imaginary power q,

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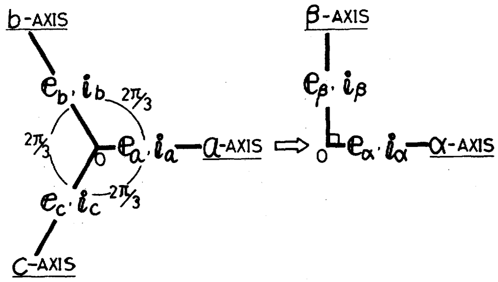


Fig. 1. α-β coordinates transformation.

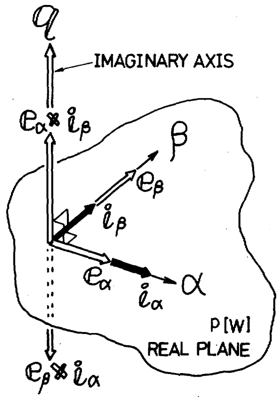


Fig. 2. Instantaneous space vectors.

which is the amplitude of $\boldsymbol{q}$, are expressed by

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}. \tag{5}$$

In (5) $e_\alpha \cdot i_\alpha$ and $e_\beta \cdot i_\beta$ obviously mean instantaneous power because they are defined by the product of the instantaneous voltage in one axis and the instantaneous current in the same axis. Therefore, p is the real power in the three-phase circuit, and its dimension is [W]. Conversely, $e_\alpha \cdot i_\beta$ and $e_\beta \cdot i_\alpha$ are not instantaneous power because they are defined by the product of the instantaneous voltage in one axis and the instantaneous current not in the same axis but in the perpendicular axis. Accordingly, q cannot be dealt with as a conventional electrical quantity. So a new dimension must be introduced for q, because the dimension is not [W], [VA], or [var]. Hereinafter, the authors have named the conventional instantaneous power p as "instantaneous real power" to distinguish the conventional instantaneous power from the instantaneous imaginary power.

Definition and Physical Meaning of Instantaneous Reactive Power

Equation (5) is changed into the following equation:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix}^{-1} \begin{bmatrix} p \\ q \end{bmatrix}. \tag{6}$$

Note that the determinant with respect to e_α and e_β in (6) is not zero.

The instantaneous currents on the α-β coordinates, i_α and i_β are divided into two kinds of instantaneous current components, respectively:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix}^{-1} \begin{bmatrix} p \\ 0 \end{bmatrix} + \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ q \end{bmatrix} = \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix} \tag{7}$$

where

α-axis instantaneous active current: $i_{\alpha p} = \dfrac{e_\alpha}{e_\alpha^2 + e_\beta^2} p$

α-axis instantaneous reactive current: $i_{\alpha q} = \dfrac{-e_\beta}{e_\alpha^2 + e_\beta^2} q$

β-axis instantaneous active current: $i_{\beta p} = \dfrac{e_\beta}{e_\alpha^2 + e_\beta^2} p$

β-axis instantaneous reactive current: $i_{\beta q} = \dfrac{e_\alpha}{e_\alpha^2 + e_\beta^2} q.$

Both the physical meaning and reason for the naming of the instantaneous active and reactive currents are clarified in the following.

Let the instantaneous powers in the α axis and the β axis be p_α and p_β, respectively. They are given by the conventional definition as follows:

$$\begin{bmatrix} p_\alpha \\ p_\beta \end{bmatrix} = \begin{bmatrix} e_\alpha i_\alpha \\ e_\beta i_\beta \end{bmatrix} = \begin{bmatrix} e_\alpha i_{\alpha p} \\ e_\beta i_{\beta p} \end{bmatrix} + \begin{bmatrix} e_\alpha i_{\alpha q} \\ e_\beta i_{\beta q} \end{bmatrix}. \tag{8}$$

The instantaneous real power in the three-phase circuit p is given as follows, using (7) and (8):

$$p = p_\alpha + p_\beta = \frac{e_\alpha^2}{e_\alpha^2 + e_\beta^2} p + \frac{e_\beta^2}{e_\alpha^2 + e_\beta^2} p + \frac{-e_\alpha e_\beta}{e_\alpha^2 + e_\beta^2} q + \frac{e_\alpha e_\beta}{e_\alpha^2 + e_\beta^2} q. \tag{9}$$

Note that the sum of the third and fourth terms of the right-hand side in (9) is always zero. From (8) and (9) the following equations are obtained:

$$p = e_\alpha i_{\alpha p} + e_\beta i_{\beta p} \triangleq p_{\alpha p} + p_{\beta p} \tag{10}$$

$$0 = e_\alpha i_{\alpha q} + e_\beta i_{\beta q} \triangleq p_{\alpha q} + p_{\beta q} \tag{11}$$

where

α-axis instantaneous active power: $p_{\alpha p} = \dfrac{e_\alpha^2}{e_\alpha^2 + e_\beta^2} p$

α-axis instantaneous reactive power: $p_{\alpha q} = \dfrac{-e_\alpha e_\beta}{e_\alpha^2 + e_\beta^2} q$

β-axis instantaneous active power: $p_{\beta p} = \dfrac{e_\beta^2}{e_\alpha^2 + e_\beta^2} p$

β-axis instantaneous reactive power: $p_{\beta q} = \dfrac{e_\alpha e_\beta}{e_\alpha^2 + e_\beta^2} q.$

Inspection of (10) and (11) leads to the following essential conclusions.

1) The sum of the instantaneous powers, $p_{\alpha p}$ and $p_{\beta p}$ coincides with the instantaneous real power in the three-phase circuit. Therefore, $p_{\alpha p}$ and $p_{\beta p}$ are named instantaneous active power.

2) The instantaneous powers, $p_{\alpha q}$ and $p_{\beta q}$ cancel each other and make no contribution to the instantaneous power flow from the source to the load. Therefore, $p_{\alpha q}$ and $p_{\beta q}$ are named instantaneous reactive power.

Note that the physical meaning of the instantaneous imaginary power defined in the three-phase circuit is quite different from that of the instantaneous reactive power in each phase.

Fig. 3 shows a generalized instantaneous power flow in a static power converter system such as a three-phase-to-three-phase cycloconverter. As shown in this figure, the instantaneous reactive powers, $p_{\alpha q}$ and $p_{\beta q}$ on the input side, are the instantaneous powers circulating between the source and the static power converter, $p_{\alpha q}'$ and $p_{\beta q}'$ are the instantaneous powers circulating between the static power converter and the load. Consequently, there is no relation in the instantaneous reactive powers on the input and output sides. The following relationship exists between the instantaneous imaginary power on the input side q and the instantaneous imaginary power on the output side q':

$$q \neq q'.$$

Assuming that there are neither energy storage components nor losses in the static power converter, the following relationship exists:

$$p = p'.$$

Furthermore, it is evident that both the instantaneous real power and the instantaneous imaginary power in a balanced sinusoidal three-phase circuit become constant. Necessarily, the instantaneous real power coincides with three times the conventional active power per one phase. In addition, the instantaneous imaginary power is numerically equal to three times the conventional reactive power per one phase. However, the instantaneous imaginary power is quite different in definition and physical meaning from the conventional reactive power based on the average value concept. The instantaneous reactive power theory, including zero-phase sequence components, is discussed in the Appendix.

CONTROL STRATEGY

Fig. 4 shows a basic compensation scheme of the instantaneous reactive power. Where p_S and q_S are the instantaneous real and imaginary powers on the source side, p_C and q_C are those on the compensator side, p and q are those on the load side.

The instantaneous reactive power compensator proposed in this paper eliminates the instantaneous reactive powers on the source side, which are caused by the instantaneous imaginary power on the load side. The compensator consists of only switching devices without energy storage components, because p_c is always zero. From (6), the instantaneous compensating currents on the α-β coordinates, $i_{C\alpha}$ and $i_{C\beta}$ are given by

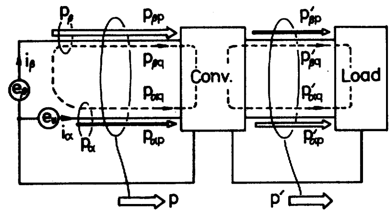


Fig. 3. Instantaneous power flow.

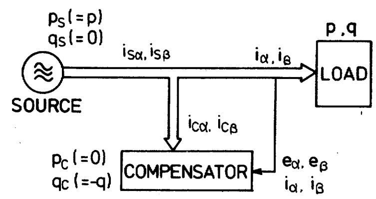


Fig. 4. Basic compensation scheme.

$$\begin{bmatrix} i_{C\alpha} \\ i_{C\beta} \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -q \end{bmatrix}. \tag{14}$$

The basic principle of the compensator will now be considered, concerning the α axis instantaneous current on the load side. The instantaneous active and reactive currents are divided into the following two kinds of instantaneous currents, respectively:

$$i_\alpha = \frac{e_\alpha}{e_\alpha^2 + e_\beta^2}\bar{p} + \frac{e_\alpha}{e_\alpha^2 + e_\beta^2}\tilde{p} + \frac{-e_\beta}{e_\alpha^2 + e_\beta^2}\bar{q} + \frac{-e_\beta}{e_\alpha^2 + e_\beta^2}\tilde{q} \tag{15}$$

where $\bar{p}$ and $\tilde{p}$ are the dc and ac components of the instantaneous real power and $\bar{q}$ and $\tilde{q}$ are the dc and ac components of the instantaneous imaginary power. The first term of the right-hand side of (15) is the instantaneous value of the conventional fundamental reactive current. The third term is the instantaneous value of the conventional fundamental reactive current. The second term is the instantaneous value of the harmonic currents which represents the ac component of the instantaneous real power. The fourth term is the instantaneous value of the harmonic currents which represents the ac component of the instantaneous imaginary power. Accordingly, the sum of the second and fourth terms is the instantaneous value of the conventional harmonic currents. Note that the second and fourth terms include a conventional negative sequency component.

Equation (15) leads to the following essential conclusions.

1) The instantaneous reactive power compensator eliminates both the third and fourth terms. For this reason, the displacement factor is unity not only in steady states but also in transient states.

2) The harmonic currents represented by the fourth term

can be eliminated by the compensator comprising switching devices without energy storage components.

EXPERIMENTAL RESULTS

Instantaneous Reactive Power Compensator System

Fig. 5 shows the experimental system. The instantaneous reactive power compensator consists of six power transistors, six power diodes, a 5-μF dc capacitor, three 0.5-μF filter capacitors, and three 2.4-mH filter reactors. The filter capacitors and reactors have the function of suppressing the harmonic currents caused by the switching operation of the power transistors. As shown in this figure, the compensator has no devices except for the 5-μF capacitor on the dc side. The 0.5-μF capacitors, 2.4-mH reactors, and the 5-μF capacitor are not used as energy storage components but are necessary for the switching operation of the power transistors. Accordingly, the higher the switching frequency of the power transistors becomes, the less the capacity of the capacitors and reactors. The maximum switching frequency is set to about 30 kHz in the following experiments.

Fig. 6 shows the control circuit of the compensator. The references of the compensating currents i_{Ca}^*, i_{Cb}^*, and i_{Cc}^* are calculated instantaneously without any time delay by using the instantaneous voltages and currents on the load side. The control circuit consists of several analog multipliers, dividers, and operational amplifiers. Note that neither low-pass filters nor integrators exist in the control circuit.

Fig. 7 shows the switching scheme of the power transistors. Now assume that the reference compensating current i_{Ca}^* is positive. The transistor Tr_1 is turned on when i_{Ca} is equal to the lower limit of i_{Ca}^*. On the contrary, Tr_1 is turned off when i_{Ca} is equal to the upper limit of i_{Ca}^*. The switching operation of the power transistors automatically forces the compensating currents i_{Ca}, i_{Cb}, and i_{Cc} to follow the reference compensating currents i_{Ca}^*, i_{Cb}^*, and i_{Cc}^*. Therefore, the power circuit can be considered as a kind of three-phase current amplifier.

Compensation Results of a Bridge Converter and a Cycloconverter

Fig. 8 shows the responses to the step variation of the load resistance of the thyristor bridge converter in which the control delay angle is zero. The displacement factor on the source side is unity not only in the steady state but also in the transient state. Fig. 9 shows the harmonic spectrum of i_{Sa} and i_a in the steady state. Although the harmonic currents of i_{Sa} are reduced considerably, i_{Sa} is not purely sinusoidal. This is due to the existence of the harmonic currents caused by the ac component of the instantaneous real power on the load side. The loss in the compensator is about 20 W, when the dc output current of the bridge converter is 10 A.

Fig. 10 shows the transient compensation results of a three-phase-to-three-phase naturally commutated cycloconverter with a balanced load. The operating conditions of the cycloconverter are as follows:

input frequency	$f = 50$ Hz
output frequency	$f_0 = 5$ Hz
output voltage ratio	$r = 0.8$.

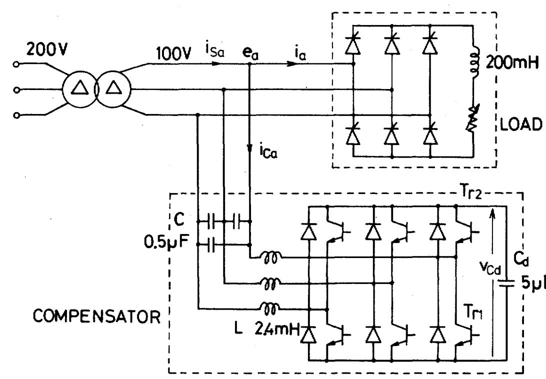
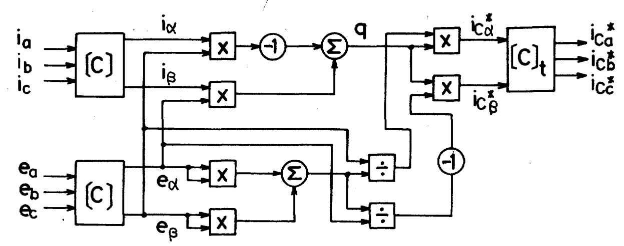


Fig. 5. Experimental system.



$$[C] = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}$$

Fig. 6. Control circuit.

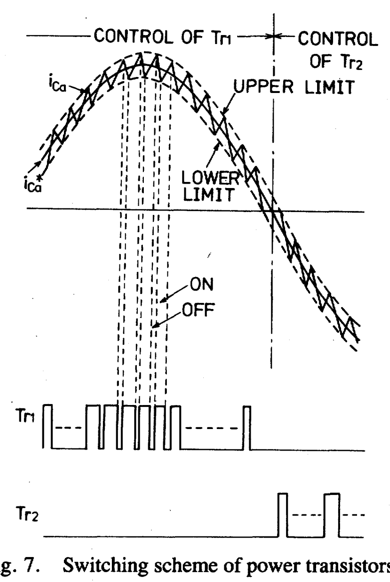


Fig. 7. Switching scheme of power transistors.

The displacement factor on the source side is unity in the transient state. Fig. 11 shows the harmonic spectrum of i_{Sa} and i_a in the steady state. The amplitude of the fundamental current of i_{Sa} is smaller than that of i_a because the fundamental reactive current of i_{Sa} is eliminated by the instantaneous reactive power compensator. Moreover, the harmonic currents having the frequencies of $f \pm 6f_o$; that is, 20 Hz and 80 Hz are eliminated because they are the harmonic currents caused by the ac component of the instantaneous imaginary power on the load side.

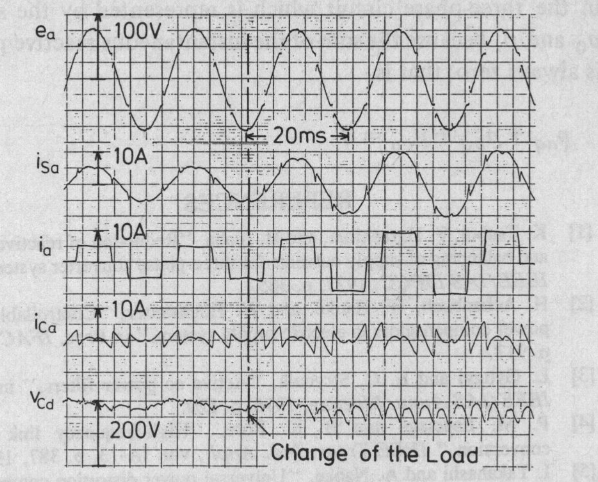


Fig. 8. Compensation of bridge converter.

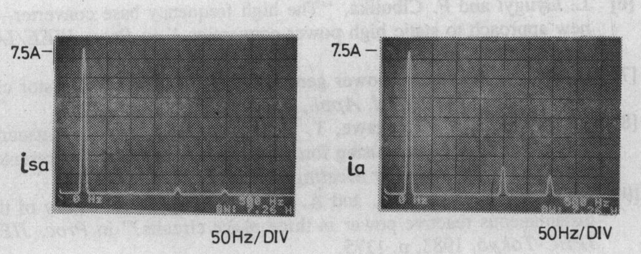


Fig. 9. Spectrum of i_{Sa} and i_a.

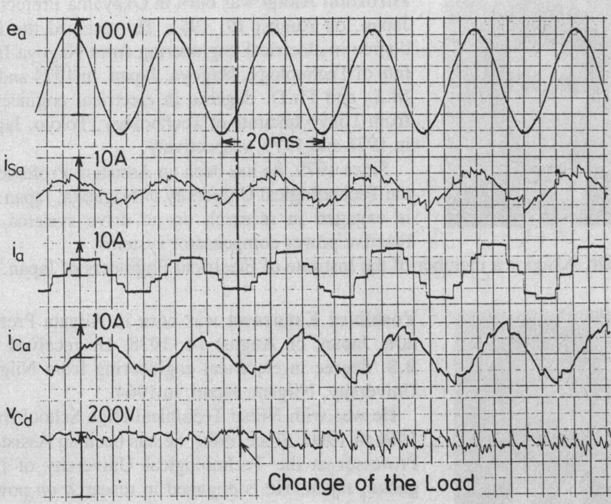


Fig. 10. Compensation of cycloconverter.

Buildup of the 5-μF DC Capacitor Voltage

The compensator operates as a three-phase diode bridge converter, while all the power transistors remain turned off. In this case, the voltage across the 5-μF dc capacitor v_{Cd} is charged up to about 140 V. The rms value of the line-to-line voltage is 100 V.

According to the experimental results, v_{Cd} builds up from 140 V to 150 ~ 200 V at the instant that the compensator comes into operation. Note that the instantaneous real power on the compensator side p_C is held at zero and that the compensator has no devices except for the capacitor on the dc side. The reason why the capacitor voltage builds up is that the switching operation of the power transistors is accompanied by the active power, corresponding to the losses in the compensator.

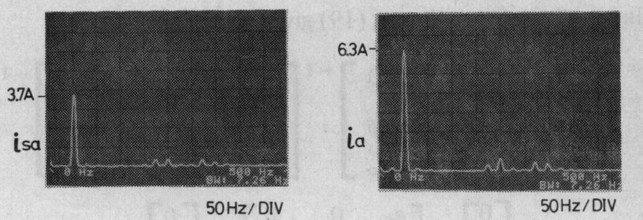


Fig. 11. Spectrum of i_{Sa} and i_a.

CONCLUSION

In this paper, the instantaneous imaginary power was introduced on the same basis as the conventional instantaneous real power in three-phase circuits. The instantaneous reactive power was defined, and the physical meaning was discussed in detail.

The instantaneous reactive power compensator comprising switching devices, which requires practically no energy storage components, was proposed, according to the theory of the instantaneous reactive power. It was verified by experiments that not only the fundamental reactive power in transient states but also the harmonic currents caused by the instantaneous imaginary power can be eliminated.

APPENDIX
DISCUSSION ON ZERO-PHASE SEQUENCE COMPONENTS

It is possible to extend the instantaneous reactive power theory developed in this paper to the three-phase circuit including zero-phase sequence components [9]. The instantaneous space vectors e_a, e_b, and e_c are transformed into 0-α-β coordinates as follows:

$$\begin{bmatrix} e_0 \\ e_\alpha \\ e_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix}. \tag{16}$$

Likewise, the instantaneous space vectors i_0, i_α, and i_β on the 0-α-β coordinates are given as follows:

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}. \tag{17}$$

The authors introduce another instantaneous power p_0, which is defined by the instantaneous space vectors, e_0 and i_0 on the 0 axis:

$$p_0 = e_0 \cdot i_0. \tag{18}$$

In (18) p_0 has been named "instantaneous zero-phase sequence power." From (3), (4), and (18), the three independent quantities p_0, p, and q are given by

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \begin{bmatrix} e_0 & 0 & 0 \\ 0 & e_\alpha & e_\beta \\ 0 & -e_\beta & e_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix}. \tag{19}$$

Inverse transformation of (19) gives

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} e_0 & 0 & 0 \\ 0 & e_\alpha & e_\beta \\ 0 & -e_\beta & e_\alpha \end{bmatrix}^{-1} \begin{bmatrix} p_0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} e_0 & 0 & 0 \\ 0 & e_\alpha & e_\beta \\ 0 & -e_\beta & e_\alpha \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ p \\ 0 \end{bmatrix} + \begin{bmatrix} e_0 & 0 & 0 \\ 0 & e_\alpha & e_\beta \\ 0 & -e_\beta & e_\alpha \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ q \end{bmatrix}$$

$$\triangleq \begin{bmatrix} i_0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + \begin{bmatrix} 0 \\ i_{\alpha q} \\ i_{\beta q} \end{bmatrix}. \tag{20}$$

From (20), the instantaneous currents on the *a-b-c* coordinates are divided in the following three components, respectively:

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{\tfrac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1 & 0 \\ 1/\sqrt{2} & -1/2 & \sqrt{3}/2 \\ 1/\sqrt{2} & -1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_0 \\ 0 \\ 0 \end{bmatrix} + \sqrt{\tfrac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1 & 0 \\ 1/\sqrt{2} & -1/2 & \sqrt{3}/2 \\ 1/\sqrt{2} & -1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 0 \\ i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + \sqrt{\tfrac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 1/2 & \sqrt{3}/2 \\ 1/\sqrt{2} & -1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 0 \\ i_{\alpha q} \\ i_{\beta q} \end{bmatrix}$$

$$\triangleq \begin{bmatrix} i_{a0} \\ i_{b0} \\ i_{c0} \end{bmatrix} + \begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix} + \begin{bmatrix} i_{aq} \\ i_{cq} \\ i_{cq} \end{bmatrix} \tag{21}$$

instantaneous zero-phase sequence current — instantaneous active current — instantaneous reactive current

where

$$i_{a0} = i_{b0} = i_{c0} = i_0/\sqrt{3}.$$

Let the *a*-, *b*-, and *c*-phase instantaneous powers be p_a, p_b, and p_c, respectively. By applying (21), the following is obtained:

$$\begin{bmatrix} p_a \\ p_b \\ p_c \end{bmatrix} = \begin{bmatrix} e_a i_{a0} \\ e_b i_{b0} \\ e_c i_{c0} \end{bmatrix} + \begin{bmatrix} e_a i_{ap} \\ e_b i_{bp} \\ e_c i_{cp} \end{bmatrix} + \begin{bmatrix} e_a i_{aq} \\ e_b i_{bq} \\ e_c i_{cq} \end{bmatrix}$$

$$\triangleq \begin{bmatrix} p_{a0} \\ p_{b0} \\ p_{c0} \end{bmatrix} + \begin{bmatrix} p_{ap} \\ p_{bp} \\ p_{cp} \end{bmatrix} + \begin{bmatrix} p_{aq} \\ p_{bq} \\ p_{cq} \end{bmatrix}. \tag{22}$$

instantaneous zero-phase sequence power — instantaneous active power — instantaneous reactive power

The instantaneous reactive powers in each phase p_{aq}, p_{bq}, and p_{cq} make no contribution to the instantaneous power flow in the three-phase circuit which is represented by the sum of p_0 and p, because the sum of the instantaneous reactive powers is always zero; that is,

$$p_{aq} + p_{bq} + p_{cq} = 0. \tag{23}$$

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