# GENERALIZED THEORY OF THE INSTANTANEOUS REACTIVE POWER IN THREE-PHASE CIRCUITS

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Abstract

The reactive power in single-phase or three-phase circuits has been defined on the basis of the average value concept. The authors propose here a new definition of the instantaneous reactive power in three-phase circuits based on the instantaneous voltages and currents, and clarify the physical meaning from the viewpoint of an instantaneous power flow. Moreover, this paper shows that not only a fundamental reactive current but also harmonic currents can be eliminated by the new instantaneous reactive power compensator without any energy storage components.

## INTRODUCTION

Various types of reactive power compensators have been developed and have come into practical use to provide power factor correction and to regulate a transmission or distribution voltage.(1)-(3) In order to compensate the reactive power instantaneously, it is necessary to define the instantaneous value of the reactive power, or the instantaneous reactive power. The instantaneous reactive power in a single-phase circuit has been defined hitherto, and the definition has been applied to a three-phase circuit. However, the definition includes the average value concept between two arbitrary consecutive times.(4) Therefore, the instantaneous reactive power can not be measured instantaneously as a unique value.

In this paper, the instantaneous imaginary power, which is a new electrical quantity, is introduced, and then the instantaneous reactive power at any instant is defined instantaneously as a unique value, using only the instantaneous values of the voltages and currents at the instant. Moreover, the instantaneous reactive power can be defined for arbitrary three-phase voltage and current waveforms without any restriction. However, it is impossible to apply this theory to a single-phase circuit.

It has been generally considered that what can be fully eliminated by a static reactive power compensator comprising switching devices without any energy storage components is only a fundamental reactive power. The theory developed in this paper leads to the conclusion that the compensator can eliminate both the fundamental reactive current and harmonic currents. The compensation principle is verified by comparison with the results of digital computer simulation.

## INSTANTANEOUS IMAGINARY POWER AND INSTANTANEOUS REACTIVE POWER

The instantaneous imaginary power and the instantaneous reactive power in a three-phase circuit excluding zero-phase sequence components are defined, and their physical meanings are clarified in the following.

**Definition of Instantaneous Imaginary Power**

Let $e_a$, $e_b$, and $e_c$ be the instantaneous values of the three-phase voltages and let $i_a$, $i_b$, and $i_c$ be those of the three-phase currents. Transformation of the instantaneous values of the three-phase voltages and currents into the $\alpha-\beta$ coordinates gives the following expression:

$$\begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (1) \qquad \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (2)$$

Fig.1 shows the instantaneous space vectors, $\mathbf{e}_\alpha$, $\mathbf{e}_\beta$, $\mathbf{i}_\alpha$, and $\mathbf{i}_\beta$ on the $\alpha-\beta$ coordinates. By using these vectors, the instantaneous power in the three-phase circuit can be defined as follows:

$$p = \mathbf{e}_\alpha \cdot \mathbf{i}_\alpha + \mathbf{e}_\beta \cdot \mathbf{i}_\beta \quad (3)$$

Since $\mathbf{e}_\alpha$ is parallel to $\mathbf{i}_\alpha$, and $\mathbf{e}_\beta$ to $\mathbf{i}_\beta$ respectively, the equation (3) is expressed as follows:

$$p = e_\alpha i_\alpha + e_\beta i_\beta \quad (4)$$

It is evident that p becomes equal to the conventional expression: $e_a i_a + e_b i_b + e_c i_c$.

In order to define the instantaneous reactive power, the authors introduce a new instantaneous space vector defined by the following expression.

$$\mathbf{q} = \mathbf{e}_\alpha \times \mathbf{i}_\beta + \mathbf{e}_\beta \times \mathbf{i}_\alpha \quad (5)$$

The vector, $\mathbf{q}$ is perpendicular to the plane of the $\alpha-\beta$ coordinates, to be faced in compliance with a right-hand rule, and $\mathbf{e}_\alpha$ is perpendicular to $\mathbf{i}_\beta$, and $\mathbf{e}_\beta$ to $\mathbf{i}_\alpha$, respectively. Therefore, the amplitude of $\mathbf{q}$, that is, q is expressed as follows:

$$q = e_\alpha i_\beta - e_\beta i_\alpha . \quad (6)$$

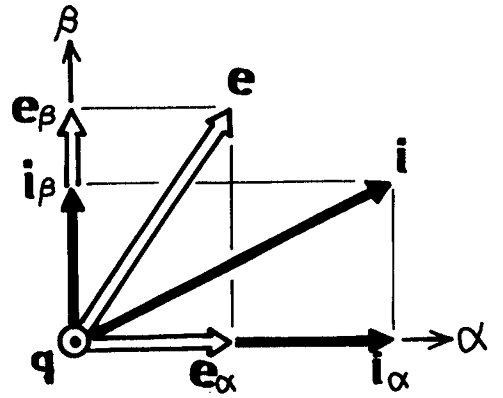


Fig.1 Instantaneous Space Vectors

The physical meaning of the first and second terms of the right-hand side in the above equation is not "instantaneous power" because of the product of the voltage in one phase and the current in the other phase. On the contrary, $e_\alpha i_\alpha$ and $e_\beta i_\beta$ in equation (4) obviously mean "instantaneous power" because of the product of the voltage in one phase and the current in the same phase. Here, the authors named the new electrical quantity defined by equation (6) "Instantaneous Imaginary Power" which is represented by the product of the instantaneous voltage and current, but can not be dealt with as a conventional quantity. Moreover, the authors named the conventional instantaneous power represented by equation (4), "Instantaneous Real Power" hereinafter, to distinguish the instantaneous real power from the instantaneous imaginary power.

According to equations (4) and (6), the instantaneous real and imaginary powers in the three-phase circuit are described by two kinds of equations, as follows:

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} i_\alpha & i_\beta \\ i_\beta & -i_\alpha \end{bmatrix} \begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} \tag{8}$$

Note that equation (7) is applicable to the voltage source and equation (8) is applicable to the current source. It is of interest that the instantaneous imaginary power is defined on a similar basis to the conventional instantaneous real power. Since the determinant with respect to $e_\alpha$ and $e_\beta$ in equation (7) is not zero, equation (7) is expressed as follows:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix}^{-1} \begin{bmatrix} p \\ q \end{bmatrix} \tag{9}$$

The instantaneous currents on the $\alpha-\beta$ coordinates, $i_\alpha$ and $i_\beta$ are determined uniquely by substituting $e_\alpha$, $e_\beta$, p, and q into equation (9). However, they can not be determined uniquely by using only $e_\alpha$, $e_\beta$, and p.

**Definition of Instantaneous Reactive Power**

The instantaneous currents in equation (9) are divided into two kinds of instantaneous current components, respectively as follows:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix}^{-1} \begin{bmatrix} p \\ 0 \end{bmatrix} + \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ q \end{bmatrix}$$

$$\triangleq \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix} \tag{10}$$

where,

$\alpha$-phase instantaneous active current: $i_{\alpha p} = \dfrac{e_\alpha}{e_\alpha^2 + e_\beta^2} p$

$\alpha$-phase instantaneous reactive current: $i_{\alpha q} = \dfrac{-e_\beta}{e_\alpha^2 + e_\beta^2} q$

$\beta$-phase instantaneous active current: $i_{\beta p} = \frac{e_\beta}{e_\alpha^2 + e_\beta^2} p$

$\beta$-phase instantaneous reactive current: $i_{\beta q} = \frac{e_\alpha}{e_\alpha^2 + e_\beta^2} q$ (11)

The first term of the right-hand side in equation (10) is named "Instantaneous Active Current", and the second term is named "Instantaneous Reactive Current". From the viewpoint of the instantaneous powers in each phase, both their physical meanings and the reason of their namings are cleared up in the following.

Let the instantaneous powers in the $\alpha$-phase and the $\beta$-phase be $p_\alpha$ and $p_\beta$, respectively. They are given by the conventional definition as follows:

$$\begin{bmatrix} p_\alpha \\ p_\beta \end{bmatrix} = \begin{bmatrix} e_\alpha & i_\alpha \\ e_\beta & i_\beta \end{bmatrix} = \begin{bmatrix} e_\alpha & i_{\alpha p} \\ e_\beta & i_{\beta p} \end{bmatrix} + \begin{bmatrix} e_\alpha & i_{\alpha q} \\ e_\beta & i_{\beta q} \end{bmatrix} \tag{12}$$

By substituing equation (11) into equation (12), the conventional instantaneous real power in the three-phase circuit, p is given as follows:

$$\begin{aligned} p &= p_\alpha + p_\beta \\ &= e_\alpha i_{\alpha p} + e_\beta i_{\beta p} + e_\alpha i_{\alpha q} + e_\beta i_{\beta q} \\ &= \frac{e_\alpha^2}{e_\alpha^2 + e_\beta^2} p + \frac{e_\beta^2}{e_\alpha^2 + e_\beta^2} p + \frac{-e_\alpha e_\beta}{e_\alpha^2 + e_\beta^2} q + \frac{e_\alpha e_\beta}{e_\alpha^2 + e_\beta^2} q \end{aligned} \tag{13}$$

It should be noticed that the sum of the third and fourth terms of the right-hand side in the above equation is always zero. From equations (12) and (13), the following equations are obtained.

$$p = e_\alpha i_{\alpha p} + e_\beta i_{\beta p} \triangleq p_{\alpha p} + p_{\beta p} \tag{14}$$

$$0 = e_\alpha i_{\alpha q} + e_\beta i_{\beta q} \triangleq p_{\alpha q} + p_{\beta q} \tag{15}$$

where,

$\alpha$-phase instantaneous active power: $p_{\alpha p} = \frac{e_\alpha^2}{e_\alpha^2 + e_\beta^2} p$

$\alpha$-phase instantaneous reactive power: $p_{\alpha q} = \frac{-e_\alpha e_\beta}{e_\alpha^2 + e_\beta^2} q$

$\beta$-phase instantaneous active power: $p_{\beta p} = \frac{e_\beta^2}{e_\alpha^2 + e_\beta^2} p$

$\beta$-phase instantaneous reactive power: $p_{\beta q} = \frac{e_\alpha e_\beta}{e_\alpha^2 + e_\beta^2} q$ (16)

Inspection of equations (14) and (15) leads to the following essential conclusions:

(1) the sum of the instantaneous powers, $p_{\alpha p}$ and $p_{\beta p}$ coincides with the conventional instantaneous real power in the three-phase circuit, p,

(2) the instantaneous powers, $p_{\alpha q}$ and $p_{\beta q}$ cancel each other, and make no contribution to the instantaneous power flow from the source to the load in the three-phase circuit.

Then, $p_{\alpha p}$ and $p_{\beta p}$ are named "Instantaneous Active Power", and $p_{\alpha q}$ and $p_{\beta q}$ are named "Instantaneous Reactive Power" *hereinafter*, as shown in equation (16). It should be noticed that the physical meaning of

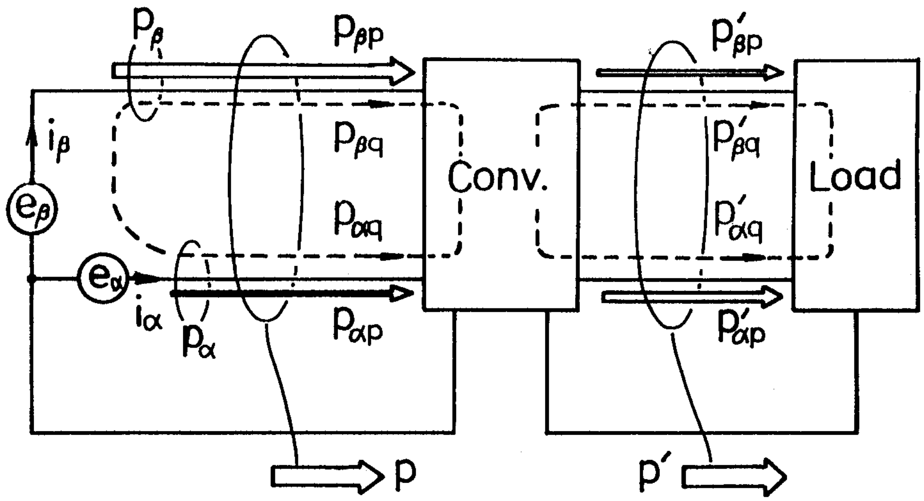


Fig.2 Instantaneous Power Flow

the instantaneous imaginary power defined in the three-phase circuit is different from that of the instantaneous reactive powers in each phase, and that the instantaneous imaginary power is the significant electrical quantity that determines uniquely the instantaneous reactive powers in each phase.

Fig.2 shows a generalized instantaneous power flow in a static power converter system such as a three-phase to three-phase cycloconverter. As shown in this figure, the instantaneous reactive powers in each phase on the input side, $p_{\alpha q}$ and $p_{\beta q}$ are the instantaneous power circulating between the source and the static power converter, and those on the load side, $p_{\alpha q}'$ and $p_{\beta q}'$ are the instantaneous power circulating between the static power converter and the load. Consequently, there is no relation in the instantaneous reactive powers on the input and output sides, and the following relationship exists between the instantaneous imaginary power on the input side, q and the instantaneous imaginary power on the output side, q'

$$q \neq q' \tag{17}$$

Assuming that there are neither energy storage components nor losses in the static power converter, the following relationship exists:

$$p = p' \tag{18}$$

Furthermore, it is evident that both the conventional instantaneous real power and the instantaneous imaginary power in a balabced sinusoidal three-phase circuit become constant. The instantaneous real power coincides with three times the conventional active power per one phase, and the instantaneous imaginary power is numerically equal to three times the conventional reactive power per one phase. However, the instantaneous imaginary power is quite different in definition and physical meaning from the conventional reactive power based on the average value concept.

## DISCUSSION ON ZERO-PHASE SEQUENCE COMPONENTS

The theory developed in the previous chapter will be extended to the three-phase circuit including zero-phase sequence components.

Transformation of the three-phase voltages, $e_a$, $e_b$, and $e_c$ to the 0-$\alpha$-$\beta$ axis voltages, $e_0$, $e_\alpha$, and $e_\beta$ gives the following:

$$\begin{bmatrix} e_0 \\ e_\alpha \\ e_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \qquad (19)$$

Likewise, the 0-$\alpha$-$\beta$ axis currents, $i_0$, $i_\alpha$, and $i_\beta$ are given as follows:

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \qquad (20)$$

where, $e_0$ and $i_0$ are zero-phase sequence components. By introducing a new transformation, the following significant equation is obtained.

$$\begin{bmatrix} p_0 \\ p \\ q \end{bmatrix} = \begin{bmatrix} e_0 & 0 & 0 \\ 0 & e_\alpha & e_\beta \\ 0 & -e_\beta & e_\alpha \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} \qquad (21)$$

In the above equation, $p_0$ is named "Instantaneous Zero-Phase Sequence Power". Inverse-transformation of equation (21) gives,

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = [C_1]^{-1} \begin{bmatrix} p \\ 0 \\ 0 \end{bmatrix} + [C_1]^{-1} \begin{bmatrix} 0 \\ p \\ 0 \end{bmatrix} + [C_1]^{-1} \begin{bmatrix} 0 \\ 0 \\ q \end{bmatrix}$$

$$\triangleq \begin{bmatrix} i_0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + \begin{bmatrix} 0 \\ i_{\alpha q} \\ i_{\beta q} \end{bmatrix} \qquad (22)$$

where,

$$[C_1] = \begin{bmatrix} e_0 & 0 & 0 \\ 0 & e_\alpha & e_\beta \\ 0 & -e_\beta & e_\alpha \end{bmatrix}$$

Substitution of equation (22) into equation (20) gives,

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = [C_2] \begin{bmatrix} i_0 \\ 0 \\ 0 \end{bmatrix} + [C_2] \begin{bmatrix} 0 \\ i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + [C_2] \begin{bmatrix} 0 \\ i_{\alpha q} \\ i_{\beta q} \end{bmatrix}$$

$$\triangleq \underbrace{\begin{bmatrix} i_{a0} \\ i_{b0} \\ i_{c0} \end{bmatrix}}_{\text{instantaneous zero-phase current}} + \underbrace{\begin{bmatrix} i_{ap} \\ i_{bp} \\ i_{cp} \end{bmatrix}}_{\text{instantaneous active current}} + \underbrace{\begin{bmatrix} i_{aq} \\ i_{bq} \\ i_{cq} \end{bmatrix}}_{\text{instantaneous reactive current}} \qquad (23)$$

where,

$$[C_2] = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1 & 0 \\ 1/\sqrt{2} & -1/2 & \sqrt{3}/2 \\ 1/\sqrt{2} & -1/2 & -\sqrt{3}/2 \end{bmatrix}$$

$$i_{a0} = i_{b0} = i_{c0} = i_0/\sqrt{3}$$

Let the a-, b-, and c-phase instantaneous powers be $p_a$, $p_b$, and $p_c$, respectively. By applying equation (23), the following equation is obtained.

$$\begin{bmatrix} p_a \\ p_b \\ p_c \end{bmatrix} = \begin{bmatrix} e_a i_{ao} \\ e_b i_{bo} \\ e_c i_{co} \end{bmatrix} + \begin{bmatrix} e_a i_{ap} \\ e_b i_{bp} \\ e_c i_{cp} \end{bmatrix} + \begin{bmatrix} e_a i_{aq} \\ e_b i_{bq} \\ e_c i_{cq} \end{bmatrix}$$

$$\triangleq \underbrace{\begin{bmatrix} p_{ao} \\ p_{bo} \\ p_{co} \end{bmatrix}}_{\text{instantaneous zero-phase power}} + \underbrace{\begin{bmatrix} p_{ap} \\ p_{bp} \\ p_{cp} \end{bmatrix}}_{\text{instantaneous active power}} + \underbrace{\begin{bmatrix} p_{aq} \\ p_{bq} \\ p_{cq} \end{bmatrix}}_{\text{instantaneous reactive power}} \qquad (24)$$

It should be noticed that the instantaneous reactive powers in each phase, $p_{aq}$, $p_{bq}$, and $p_{cq}$ make no contribution to the instantaneous power flow in the three-phase circuit which is represented by the sum of $p_0$ and p, because the sum of the instantaneous reactive powers is always zero, that is,

$$p_{aq} + p_{bq} + p_{cq} = 0 \qquad (25)$$

## INSTANTANEOUS ACTIVE AND REACTIVE POWER COMPENSATION

Fig.3 shows a basic compensation scheme of the instantaneous active and reactive power. Here, the following equation exists concerning the instantaneous real powers and the instantaneous imaginary powers.

$$\begin{bmatrix} p_c \\ q_c \end{bmatrix} = \begin{bmatrix} p^* \\ q^* \end{bmatrix} - \begin{bmatrix} p \\ q \end{bmatrix} . \qquad (26)$$

where, $p^*$ and $q^*$ are the desired instantaneous real power and desired instantaneous imaginary power on the source side, $p_c$ and $q_c$ are the instantaneous real power and instantaneous imaginary power on the compensator side, and p and q are the instantaneous real power and instantaneous imaginary power on the load side.

The control strategy of the compensator based on equation (26) can be classified into three groups.

(1) **Instantaneous Active Power Compensator**

This compensator reduces variations of the instantaneous active powers on the load side. The instantaneous compensating currents on the $\alpha$-$\beta$ coordinates, $i_{\alpha c}$ and $i_{\beta c}$ are given by

$$\begin{bmatrix} i_{\alpha c} \\ i_{\beta c} \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix}^{-1} \begin{bmatrix} p^* - p \\ 0 \end{bmatrix} \qquad (27)$$

(2) **Instantaneous Reactive Power Compensator**

This compensator eliminates the instantaneous reactive powers caused by the instantaneous imaginary power on the load side. The compensating currents are controlled by the following equation:

$$\begin{bmatrix} i_{\alpha c} \\ i_{\beta c} \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ q^* - q \end{bmatrix} \qquad (28)$$

It should be noted that the compensator can comprise in principle only

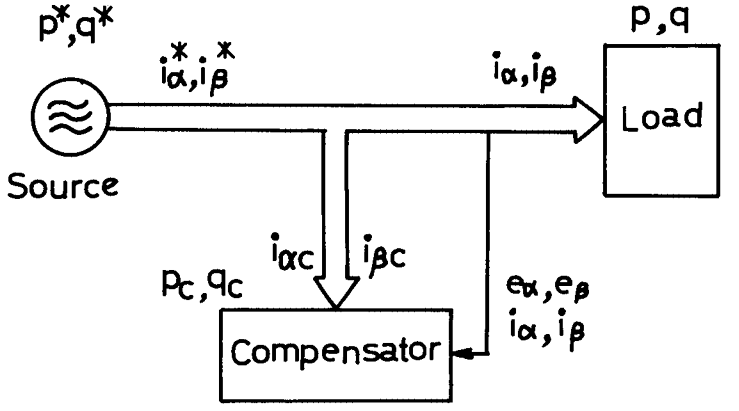


Fig.3 Basic Compensation Scheme

switching devices without any energy storage components, because $p_c$ is always zero.

(3) **Instantaneous Active and Reactive Power Compensator**

This compensator reduces variations of the instantaneous active powers and eliminates the instantaneous reactive powers. The compensating currents are controlled by the following equation:

$$\begin{bmatrix} i_{\alpha c} \\ i_{\beta c} \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix}^{-1} \begin{bmatrix} p^* - p \\ q^* - q \end{bmatrix} \qquad (29)$$

The basic principle of these above mentioned compensators is now considered on the viewpoint of compensation of the $\alpha$-phase instantaneous current on the load side, assuming sinusoidal three-phase voltage source. The instantaneous active and reactive currents in equation (11) are divided into the following two kinds of instantaneous currents, respectively.

$$i_\alpha = \frac{e_\alpha}{e_\alpha^2 + e_\beta^2}\bar{p} + \frac{e_\alpha}{e_\alpha^2 + e_\beta^2}\tilde{p} + \frac{-e_\beta}{e_\alpha^2 + e_\beta^2}\bar{q} + \frac{-e_\beta}{e_\alpha^2 + e_\beta^2}\tilde{q} \qquad (30)$$

where, $p = \bar{p} + \tilde{p}$ , $q = \bar{q} + \tilde{q}$ , and $\bar{p}$ and $\tilde{p}$ are the dc component and ac component of the instantaneous real power, and $\bar{q}$ and $\tilde{q}$ are the dc component and ac component of the instantaneous imaginary power, respectively.

The first term of the right-hand side in equation (30) is the instantaneous value of the conventional fundamental active current, and the third term is that of the conventional fundamental reactive current. The second term is that of the harmonic currents caused by the ac component of the instantaneous real power, and the fourth term is that of the ac component of the instantaneous imaginary power. Accordingly, the sum of the second and fourth terms is the instantaneous value of the conventional harmonic currents. Note that the second and fourth terms include a conventional negative sequence component. The equation (30) leads to the following essential conclusions:

(1) The instantaneous active power compensator eliminates only the second term, when $p^*$ is equal to $\bar{p}$. Therefore, the displacement factor on the source side is not improved.

(2) The instantaneous reactive power compensator eliminates both the

third and fourth terms, when $q^*$ is zero. For this reason, the displacement factor is unity in transient states as well as in steady states. Moreover, it is of interest that the harmonic currents represented by the fourth term can be eliminated by a static compensator comprising only switching devices without any energy storage components.

(3) The instantaneous active and reactive power compensator eliminates all of the second, third, and fourth terms, when $p^*$ is equal to $\overline{p}$, and $q^*$ is zero. Therefore, the three-phase currents on the source side become the balanced sinusoidal waveforms, and the power factor is unity.

## DIGITAL SIMULATION RESULTS

The compensation principle of the instantaneous reactive power is verified by means of digital computer simulation.

### Three-Phase Thyristor Phase-Controlled Converters

Fig.4 shows the compensation results of a three-phase thyristor phase-controlled converter, in which the control delay angles, $\alpha$ are $0°$ and $30°$. Note that the a-phase instantaneous current on the load side, $i_a^*$ coincides with the a-phase instantaneous active current on the load side because of assuming an ideal instantaneous reactive power compensator. As shown in this figure, $i_a^*$ is not purely sinusoidal, due to the harmonic current represented by the second term in equation (30). However, the displacement factor is unity, regardless of the control delay angle.

Fig.5 shows the calculation results of the 5th, 7th, 11th, and

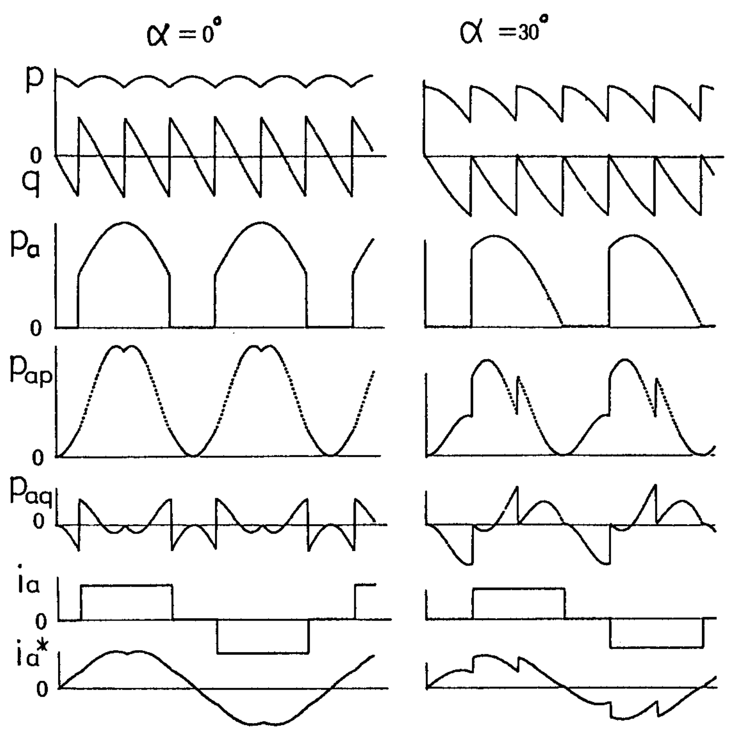


Fig.4 Compensation Results of the Thyristor Phase-Controlled Converter

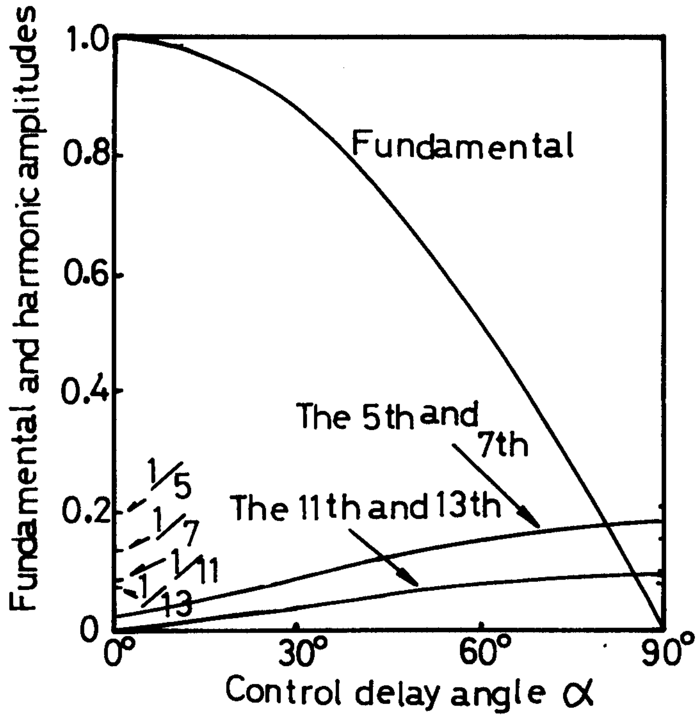


Fig.5 Calculation Results of Harmonic Amplitudes of $i_a^*$

13th harmonic amplitudes of $i_a^*$ to the control delay angle. The amplitudes of the 5th and 11th harmonics are equal to those of the 7th and 13th harmonics, respectively. Each harmonic amplitude is reduced gradually with reduction of the control delay angle.

**Three-phase to Three-phase Naturally Commutated Cycloconverters**

The harmonic frequencies of the input current in a three-phase to three-phase naturally commutated cycloconverter with a balanced load, $f_h$ are given as follows:

$$f_h = f \pm 6nf_0 \;,\quad (6m\pm1)f \pm 6nf_0 \qquad ( m = 1,2,3\cdots \;,\quad n = 0,1,2\cdots)$$

where, f is the input frequency and f is the output frequency. Moreover, the harmonic amplitudes were already analyzed in detail by B.R.Pelly.

In designing the compensator of the harmonic currents, it is important to analyze theoretically whether they are the second term in equation (30), or the fourth term. The reason why it is important is that the harmonic analysis determines whether energy storage components are essentially necessary to the compensator or not.

Let $e_a$, $e_b$, and $e_c$ be the balanced three-phase sinusoidal voltages:

$$e_a = E_m \sin\omega t \;,\quad e_b = E_m \sin(\omega t - 2\pi/3) \;,\quad e_c = E_m \sin(\omega t - 4\pi/3) \qquad (31)$$

where, $E_m$ is the amplitude of the phase voltage and $\omega$ is equal to $2\pi f$. Assuming the commutating overlap angle to be negligible, the instantaneous currents of the harmonic family having the frequencies of $f \pm 6nf_0$ that is, $i_{af}$, $i_{bf}$, and $i_{cf}$ are given as follows:

$$i_{af} = \frac{6\sqrt{3}}{\pi}\left[A_0 \sin\omega t + \left\{ B_0 + \sum_{n=1,2,\cdots}\left( B_{sn} \sin 6n\omega_0 t + B_{cn} \cos 6n\omega_0 t \right)\right\} \cos\omega t\right]$$

$$i_{bf} = \frac{6\sqrt{3}}{\pi}\left[A_0 \sin(\omega t - \frac{2}{3}\pi) + \left\{B_0 + \sum_{n=1,2,\cdots}(B_{sn}\sin 6n\omega_o t + B_{cn}\cos 6n\omega_o t)\right\}\cos(\omega t - \frac{2}{3}\pi)\right]$$

$$i_{cf} = \frac{6\sqrt{3}}{\pi}\left[A_0 \sin(\omega t - \frac{4}{3}\pi) + \left\{B_0 + \sum_{n=1,2,\cdots}(B_{sn}\sin 6n\omega_o t + B_{cn}\cos 6n\omega_o t)\right\}\cos(\omega t - \frac{4}{3}\pi)\right] \quad (32)$$

where, $A_0$, $B_0$, $B_{sn}$, and $B_{cn}$ are the constants and $\omega_o$ is equal to $2\pi f_0$. By substituting equations (31) and (32) into equations (1) and (2), and then into equation (7), $p_f$ and $q_f$ are obtained.

$$p_f = \frac{9\sqrt{3}}{\pi} E_m A_0$$

$$q_f = -\frac{9\sqrt{3}}{\pi} E_m\left\{B_0 + \sum_{n=1,2,\cdots}(B_{sn}\sin 6n\omega_o t + B_{cn}\cos 6n\omega_o t)\right\} \quad (33)$$

The above equation leads to the following significant conclusions:

(1) The instantaneous real power is constant. Conversely, the instantaneous imaginary power is fluctuating at the frequencies of $6nf_o$.

(2) The harmonic currents having the frequencies of $f \pm 6nf_0$ are not the second term in equation (30), but the fourth term.

(3) Therefore, they can be fully eliminated by the instantaneous reactive power compensator comprising only switching devices without any energy storage components.

According to similar analysis, it is, however, concluded that the harmonic currents having the frequencies of $(6mf \pm 1)f \pm 6nf_0$ can be eliminated by the above mentioned compensator, because both the instantaneous real and imaginary powers are fluctuating at the frequencies of the $6mf \pm 6nf_0$.

Fig.6 shows the compensation results of the cycloconverter by means of the instantaneous reactive power compensator discussed above. The simulation conditions are as follows:

| | |
|---|---|
| input frequency: | $f = 50$ Hz |
| output frequency: | $f_o = 5$ Hz |
| displacement factor of load: | $\cos\theta = 0.8$ |
| output voltage ratio: | $r = 0.8$ |

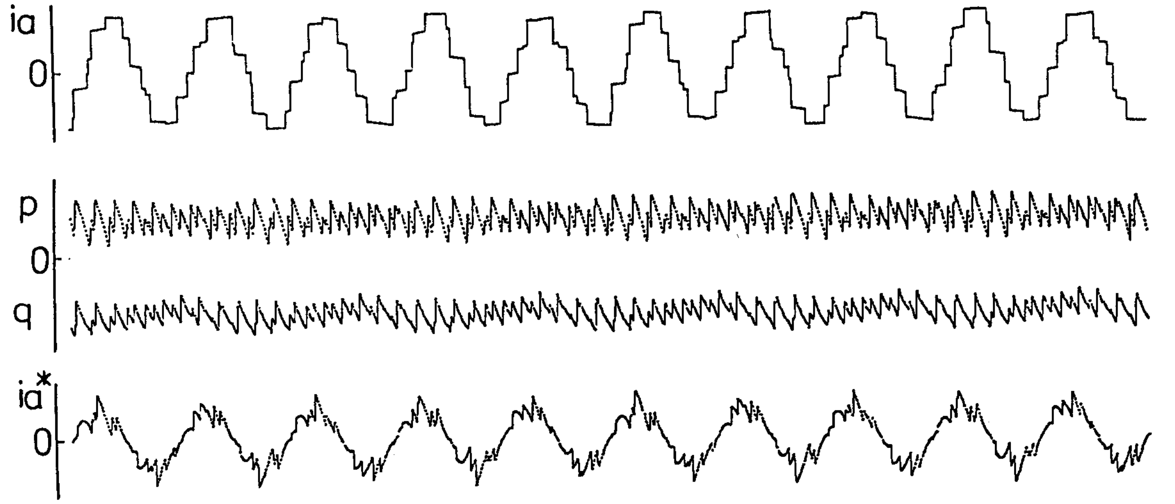


Fig.6 Compensation Results of the Cycloconverter

The harmonic components having the frequencies of $6nf_0$ exist in the instantaneous imaginary power, although the amplitudes of them are considerably small. On the other hand, they do not exist in the instantaneous real power. Moreover, the waveform of the a-phase instantaneous current on the source side, $i_a^*$ shows that it is possible to detect and calculate without any time delay not only the fundamental reactive current but also the harmonic currents represented by the fourth term in equation (30), which have the frequencies of $f \pm 6nf_0$ and $(6mf \pm 1)f \pm 6nf_0$, and to eliminate them fully.

## CONCLUSION

In this paper, the new concept of the instantaneous imaginary power was introduced on the same basis as the conventional instantaneous (real) power in three-phase circuits. The instantaneous reactive powers in each phase were defined at any instant, using only the instantaneous voltages and currents, and their physical meanings were discussed in detail.

According to the generalized theory of the instantaneous reactive power developed in this paper, the conventional harmonic currents were divided into two kinds caused by the instantaneous (real) power and the instantaneous imaginary power. It was clarified that not only the fundamental reactive current but also the latter harmonic currents could be fully eliminated by the instantaneous reactive power compensator comprising swtching devices without any energy storage components.

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