THE PARAMETRON, A DIGITAL COMPUTING ELEMENT WHICH UTILIZES PARAMETRIC OSCILLATION

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The Parametron, a Digital Computing Element which Utilizes Parametric Oscillation*

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Summary—The following is a brief description of the basic principles and applications of the parametron, which is a digital computer element invented by the author in 1954. A parametron element is essentially a resonant circuit with a nonlinear reactive element which oscillates at one-half the driving frequency. The oscillation is used to represent a binary digit by the choice between two stationary phases π radians apart. The basic principle of logical circuits using the parametron is explained, and research on and applications of parametrons in Japan are described.

I. Introduction

IN keeping with the remarkable progress of electronic computers in recent years, studies on digital computing elements and memory devices have been energetically conducted in various laboratories. Among them, one will find new applications of physical phenomena and effects that have never before been utilized in the field of electronics; the cryotron, which uses superconductivity, and the spin echo memory are typical examples.

In 1954 the author discovered that a phenomenon called parametric oscillation, which had been known for many years, can be utilized to perform logical operations and memory functions, and gave the name "Parametron" to the new digital component made on this principle [1], [21]–[23].

A digital computing circuit made of parametrons may consist only of capacitors, ferrite-core coils and resistors, while diodes and rectifiers may be dispensed with. The parametron, therefore, is considered to be extremely sturdy, stable, durable, and inexpensive. Owing to these advantages, intensive studies have started in several laboratories in Japan to apply parametrons to various digital systems. At present, nearly half of the Japanese electronic computers in operation use parametrons for logical elements. Further applications have been made to such devices as telegraphic equipments, telephone switching systems and numerical control of machine tools.

Parametric oscillation, from which the name "Parametron" derives, is not a unfamiliar phenomenon—a playground swing and Melde's experiment are examples of parametric oscillations in mechanical systems.

In order to drive a swing, the rider bends and then straightens his body and thereby changes the length l between the center of gravity of his body and the fulcrum of the ropes. The swing is a mechanical resonant system and its resonant frequency is determined by this length l and the gravitational constant g. The oscillation of the swing is energized by the periodic variation of the parameter l which determines the resonant frequency. Similarly, in Melde's experiment, shown in Fig. 1, a periodic variation is given to the tension, which is a parameter that determines the resonant frequency of the string. In this case, the exciting energy which varies the tension is supplied from a tuning fork of resonant frequency $2f$, which is twice the resonant frequency f of the string. In other words, the oscillating frequency of the string is a subharmonic equal to half the frequency of the energy source, that is, it is the second subharmonic. The mechanism of building up of this subharmonic is shown in Fig. 2. As the string moves away from the equilibrium position, the tension is weakened and the maximum amplitude increases; as the string returns to the center position, the tension is strengthened and the kinetic energy increases.

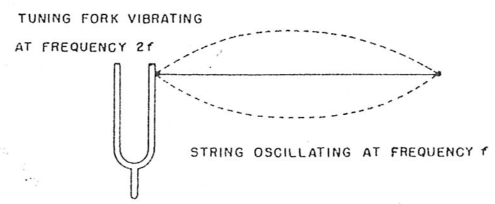

Fig. 1—Melde's experiment.

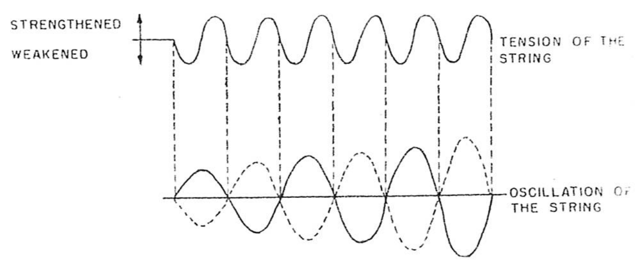

Fig. 2—Build up of oscillation of the string.

In an electrical system, inductance and capacitance are the parameters which determine the resonant frequency. Parametric oscillation therefore can be produced in a resonant circuit by periodically varying one of the reactive elements composing the resonant circuit [18].

A parametron element is essentially a resonant circuit with a reactive element varying periodically at frequency $2f$ which generates a parametric oscillation at the subharmonic frequency f. In practice, the periodic

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variation is accomplished by applying an exciting current of frequency $2f$ to a balanced pair of nonlinear reactors, such as ferrite-core coils and nonlinear capacitors made of ferroelectric material or of the barrier capacitance of semiconductor junctions.

The subharmonic parametric oscillation thus generated has a remarkable property in that the oscillation will be stable in either of two phases which differ by π radians with respect to each other. Utilizing this fact, a parametron represents and stores one binary digit, "0" or "1," by the choice between these two phases, 0 or π radians. The solid line and the dotted line in Fig. 2 illustrate the building up of these two kinds of oscillation.

Under certain resonance conditions, the oscillation generated in the parametron is "soft," that is, it is easily self-started from any small initial amplitude. In this case, the choice between the two stable phases of the oscillation having a large amplitude can be made by controlling the phases of the small initial oscillation. This fact may be regarded as amplification and its mechanism may best be understood as superregeneration with the phase of the oscillation quantized to two states. In order to make use of this effectively, quenching means are provided in parametron circuits to interrupt parametric oscillation. Besides the memory and amplifying action, parametrons can also perform various logical operations based on a majority principle by applying the algebraic sum of oscillation voltages of an odd number of parametrons to another parametron in which the algebraic sum voltage works as the small initial oscillation voltage.

Mathematical studies on parametric oscillations of small amplitude in a linear region have been conducted in detail in the past. The results will be found in textbooks on differential equations under such headings as linear differential equations with periodic coefficients, Mathieu's equation, Hill's equation, and Floque's theorem [16], [17]. However, in order to describe the actual behavior of parametrons quantitatively, one has to take nonlinearity into consideration, and this will be treated in the Appendix.

The application of parametric oscillation to amplifying electrical signals is not a new idea. We find in Peterson's patent of 1932 [29], an idea for an amplifier based on the same principle as the parametric amplifier, which is now one of the most discussed topics in the field of electronics. In a parametric amplifier, two resonant circuits, respectively tuned to signal frequency f_s and idling frequency f_i, are coupled together regeneratively through a nonlinear reactor to which is applied a voltage of pumping frequency f_p, satisfying the condition $f_p = f_i + f_s$. A parametric amplifier performs regenerative amplification of signals and may produce, as well, a pair of spontaneous oscillations at frequency f_s and f_i.

A parametron producing a subharmonic oscillation may be regarded as a degenerative case of a parametric amplifier, in which the two resonant circuits for f_s and f_i are reduced to a single common circuit, so that $f_s = f_i = f$, and $f_p = 2f$. Consequently, the basic principle of the amplifying mechanism of the parametron may be considered the same as that of the parametric amplifier. The degeneracy in the number of resonant circuits, however, makes possible the phase quantizing nature of the oscillation. While this is generally unfavorable for amplifying ordinary continuous waves, it is very useful for representing and storing a binary digit in the parametron.

Parametric oscillation of the second subharmonic mode in an electrical system has been known for many years and has been applied to frequency dividers [18]. On the other hand, the idea that two stable phases exist and can be applied to digital operations can be found only in a patent [30] of the late Professor von Neumann, so far as the author knows. Von Neumann proposed, completely independent of the author, a scheme similar to the parametron. His idea, however, seems to have not yet been developed into practical use.

If the resonance condition of a circuit which produces the subharmonic parametric oscillation is slightly altered, a "hard" oscillation, *i.e.*, not self-starting, will be produced. This circuit, generally, has three stable states, namely, "no oscillation," "oscillating at 0 phase," and "oscillating at π-radian phase." In Japan such an element is usually called a "tristable parametron," while in the case of "soft" oscillation it is called a "bistable parametron." In the hard oscillation circuit, *i.e.*, the tristable parametron, a binary digit can be represented by the presence or absence of oscillation. This scheme has also been proposed independently by Clary [31].

II. Basic Principle

The parametron is essentially a resonant circuit in which either the inductance or the capacitance is made to vary periodically. Fig. 3 shows circuit diagrams for parametron elements. The parametron element in Fig. 3(a) consists of coils wound around two magnetic ferrite toroidal cores $F1$ and $F2$, a capacitor C, a damping resistor R, and a small toroidal transformer T. Each of the cores $F1$ and $F2$ has two windings and these are connected together in a balanced configuration, one winding $L = L' + L''$ forming a resonant circuit with the capacitor C and being tuned to frequency f. An exciting current, which is a superposition of a dc bias and a radio frequency current of frequency $2f$, is applied to the other winding, $l' + l''$, causing periodic variation in the inductance $L = L' + L''$ of the resonant circuit at frequency $2f$.

The parametron in Fig. 3(b) consists of two nonlinear capacitors C' and C'' which form the resonant circuit with the inductance L. An exciting voltage of frequency $2f$ is supplied between the neutral point of the two nonlinear capacitors C', C'' and the neutral point of the inductance L, causing periodic variation in the tuning capacitance $C(1/C = 1/C' + 1/C'')$ at frequency $2f$. As

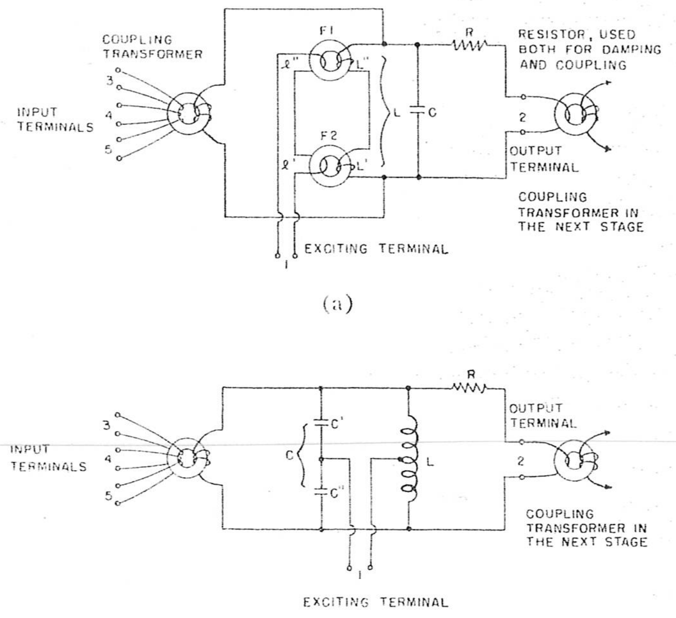

Fig. 3—Circuit diagram of parametrons, (a) Magnetic type. (b) Capacitive type.

the results are entirely analogous in both cases, the following explanations will be given only for the former case.

The operation of the parametron is based on a spontaneous generation of a second-subharmonic parametric oscillation, that is a self-starting oscillation of frequency f, in the resonant circuit. Parametric oscillation is usually treated and explained in terms of Matheiu's equation. A more intuitive explanation, however, may be obtained by the following consideration.

Let the inductance L of the resonant circuit be varied as

$$L = L_0(1 + 2\Gamma \sin 2\omega t) \tag{1}$$

where $\omega = 2\pi f$, and let us assume the presence of a sinusoidal ac current I_f in the resonant circuit at frequency f, which can be broken down into two components as follows:

$$I_f = I_s \sin(\omega t) + I_c \cos(\omega t). \tag{2}$$

Then, assuming that the rate of the variation of amplitudes of the sine and cosine components, I_s and I_c, are small compared with ω, the induced voltage V will be given by

$$\begin{aligned} V = d/dt(LI_f) &= \omega L_0(I_s \cos \omega t - I_c \sin \omega t) \\ &+ 3\Gamma\omega L_0(I_s \sin 3\omega t + I_c \cos 3\omega t) \\ &+ \Gamma\omega L_0(-I_s \sin \omega t + I_c \cos \omega t). \end{aligned} \tag{3}$$

The first term shows the voltage due to a constant inductance L_0, and the second term or the third harmonic term may be neglected in our approximation, since it is off resonance. The third term, which is essential for the generation of the second subharmonic, shows that the variable part of the inductance behaves like a negative resistance $-r = -\Gamma\omega L_0$ for the sine component I_s, but behaves like a positive resistance $+r = \Gamma\omega L_0$ for the cosine component I_c.

Therefore, provided that the circuit [Fig. 3(a)] is nearly tuned to f, the sine component I_s of any small oscillation (Ⓐ in Fig. 4), will build up exponentially (Ⓑ in Fig. 4), while its cosine component will damp out rapidly. If the circuit were exactly linear, the amplitude would continue to grow indefinitely. Actually, the nonlinear B-H curve of the cores causes detuning of the resonance circuit and hysteresis loss also increases with increasing amplitude, so that a stationary state (Ⓒ in Fig. 4) will rapidly be established, as in vacuum-tube oscillators. Details of the amplitude limiting mechanism, which is essentially a nonlinear problem, will be treated in the Appendix. The solution of the problem will be illustrated most intuitively by showing the locus of the sine and cosine components, I_s and I_c in the (I_s, I_c) plane. Fig. 5 shows an example of such loci for a typical case $\alpha = 0$, $\delta = \Gamma/2$ (*cf.* Appendix). The abscissa represents the sine component I_s and the ordinate, the cosine component I_c. If we introduce polar coordinates (R, ϕ) in the (I_s, I_c) plane, it will be seen easily from (1) that R and ϕ, respectively, indicate the instantaneous amplitude and phase of the oscillation. The saddle point at the origin indicates the exponential build up of oscillation which is in a definite phase relation to the excitation wave of frequency $2f$. Spiral points A and A' in the figure indicate the stable states of stationary oscillation. The existence of two possible phases in this oscillation which differ by π radians from each other, corresponding to A and A', should be noted. These two modes of oscillation are respectively shown by the solid line and dotted line in Fig. 4. An especially important feature is that the choice between these two modes of stationary oscillation is effected entirely by the sign of the sine component of the small initial oscillations that have existed in the circuit (A in Fig. 4). In other words, the choice between A and A' in Fig. 5 depends on which side of the thick curve B-B' (called separatrix) the point representing the initial state lies. An initial oscillation of quite small amplitude is sufficient to control the mode or the phase of stationary oscillation of large amplitude which is to be used as the output signal. Hence, the parametron has an amplifying action which may be understood as superregeneration. The upper limit of this superregenerative amplification is believed to be determined only by the inherent noise, and an amplification of as high as 100 db has been reported.[1]

The existence of dual mode of stationary oscillation can be made use of to represent a binary digit, "0" and "1" in a digital system, and thus a parametron can store

[1] A personal communication from Z. Kiyasu, of Electrical Communication Laboratory, Nippon Telephone and Telegraph Co., Tokyo, Japan.

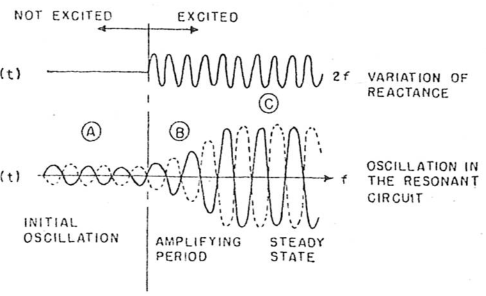

Fig. 4—Oscillation of parametrons.

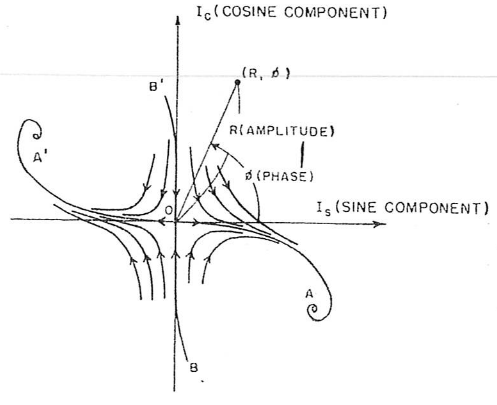

Fig. 5—The amplitude-to-phase (R, ϕ) locus of an oscillating parametron.

1 bit of information. However, oscillation of parametrons in this stationary state is extremely stable, and if one should try to change the state of an oscillating parametron from one mode to another just by directly applying a control voltage to the resonant circuit, a signal source as powerful as the parametron itself would be necessary. This difficulty can be got around by providing a means for quenching the oscillation, and making the choice between the two modes, *i.e.*, the rewriting of information, by a weak control voltage applied at the beginning of each building up period, making use of the superregenerative action.

Actually, this is done by modulating the exciting wave by a periodic square wave which also serves as the clock pulse of the computer. Hence, for each parametron there is an alternation of active and passive periods, corresponding to the switching on and off of the exciting current. Usually, the parametron device uses three clock waves, labeled I, II and III, all having the same pulse recurrence frequency, but switched on and off one after another in a cyclic manner as shown in Fig. 6. This method of exciting each of the parametrons in a digital system with either one of the three exciting waves I, II and III is usually called the "three beat" or the "three subclock" excitation.

III. Basic Digital Operations by Parametrons

Digital systems can be constructed using parametrons by intercoupling parametron elements in different

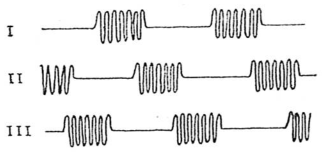

Fig. 6—The exciting current of three groups, I, II and III.

groups by a coupling element, the toroidal transformers shown in Fig. 3.

Figs. 7 to 9 show the basic parametron circuits.

The parametron is a synchronous device and operates in rhythm with the clock pulse. Each parametron takes in a new binary digit ("1" or "0") at the beginning of every active period, and transmits it to the parametrons of the next stage with a delay of one-third of the clock period. This delay can be used to form a delay line. Fig. 7 shows one such delay line which consists of parametrons simply coupled in a chain, each successive parametron element belonging each to the groups, I, II, III, I, · · · . Hence, the phase of oscillation of a parametron in the succeeding stage will be controlled by that in the preceding stage, and a binary signal x applied to the leftmost parametron will be transmitted along the chain rightwards in synchronism with the switching of the exciting currents. Hence, the circuit may be used as a delay line or a dynamic memory circuit.

Fig. 8 shows how logical operations can be performed using parametrons. In the figure, the outputs of the three parametrons X, Y and Z in stage I, which are all oscillating at a voltage V, are coupled to the single parametron U in stage II with a coupling factor k. As the effective phase control signal acting on U is the algebraic sum of the three signals from X, Y and Z, each of which has the value $+kV$ or $-kV$, the mode of U representing a binary signal u will be determined according to the majority of the three binary signals x, y and z, respectively represented by the oscillation modes of X, Y and Z. It would be possible, in principle, to generalize the majority circuit of Fig. 8(a) to 5, 7, 9, · · · inputs, that is, to any odd number of inputs. In practice, however, the nonuniformity in the characteristics of each parametron causes disparity in the input signals and makes the majority decision inaccurate, and this fact limits the allowable number of inputs to 3 or 5 in most cases.

It is easily seen that the majority operation just outlined includes the basic logical operations "and" and "or" as special cases. Suppose that one of the three inputs in Fig. 8(a), say z, is fixed to a constant value "1," then we obtain a biased majority decision on the remaining two inputs x and y, and the resulting circuit gives "x or y" as shown in Fig. 8(c). Similarly, if z is fixed to a constant value "0," we obtain a circuit for "x and y" as shown in Fig. 8(d). These constant signals are actually derived from a special parametron called constant parametron, or some other voltage source equivalent to it. If 2 out of 5 inputs in a "5-input majority operation" are made constants, either "1" or "0," we shall obtain either a "or" or "and" circuit respectively for three input variables x, y and z, as shown in Figs. 8(e) and 8(f).

"Complementation" or "not" operation can be made most simply in parametron circuits. In order to change the binary signal "1" into "0" and vice versa, we have only to reverse the polarity of the input signal, and this can be done by coupling two parametrons in reversed polarity as shown in Fig. 9. In the schematic diagram, such coupling in reversed polarity will be indicated by a short bar in the coupling line as shown in Fig. 9.

Since a digital system of any complexity can be synthesized by combining the four basic circuit elements, namely "delay," "and," "or," and "not," it will be seen that a complete digital system, *e.g.*, a general purpose electronic computer, can in principle be constructed using only one kind of circuit element—the parametron. It should be noted that the above conclusion presupposes that some means for logical branching, that is amplification of signal power, is provided. Now parametrons have a large superregenerative amplification and the output of a single parametron can supply input signals to a rather large number of parametrons in the next stage. For the parametrons currently used, the maximum allowable number of the branching is from 10 to 20. This feature adds flexibility in design of digital systems using parametrons.

IV. Simple Examples of Parametron Circuits

A complete digital apparatus may consist of hundreds or thousands of parametrons, coupled to each other by wires (via resistors and transformers) to form a network. Such a network of parametrons may be conveniently described by a schematic diagram.

At this point we will give a short summary of the rules and conventions for schematic diagrams currently in use in Japan.

Each parametron is represented by a small circle, as shown in the figures. Each pair of circles is connected by a line if corresponding parametrons are coupled, one

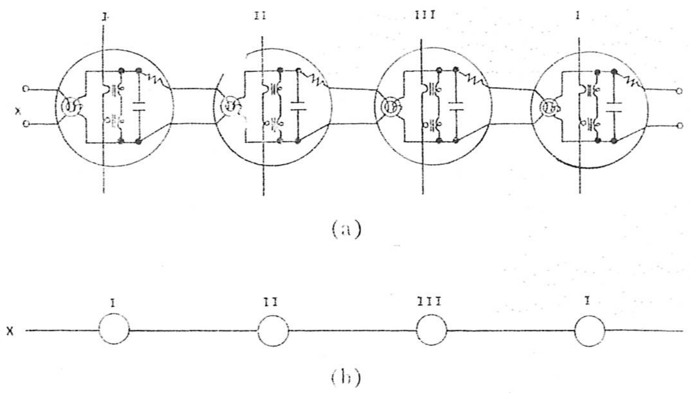

Fig. 7—A parametron delay-line circuit.

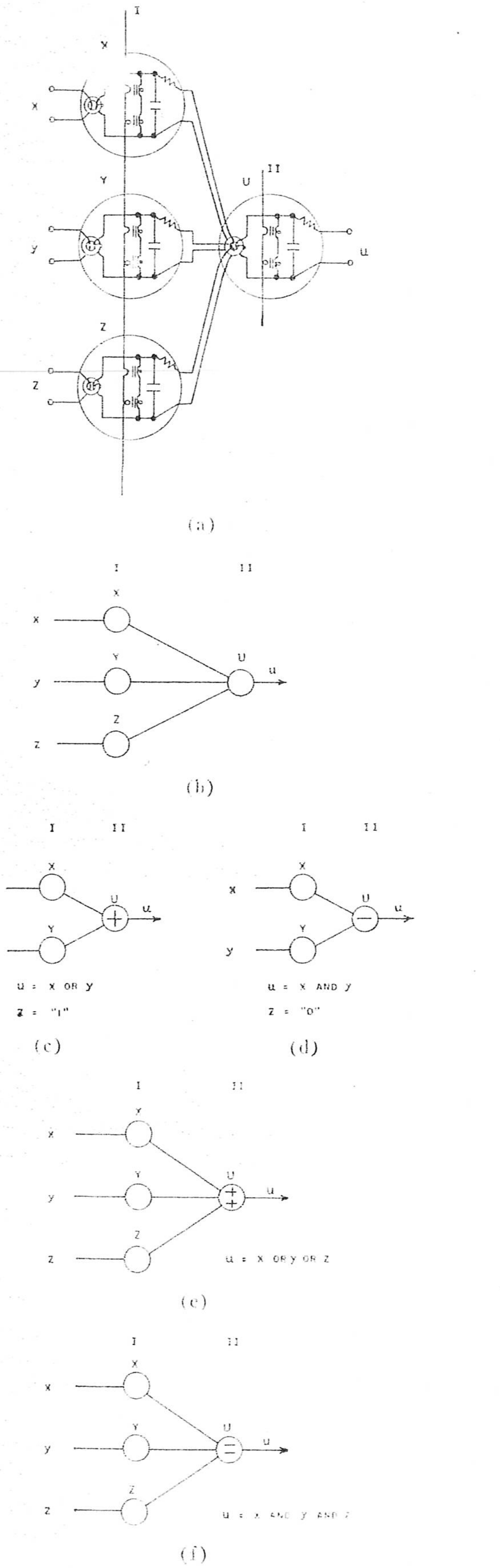

Fig. 8—A three-input majority operation circuit. (a) Circuit diagram. (b) The schematic diagram of (a). (c) An "or" circuit for two inputs. (d) An "and" circuit for two inputs. (e) An "or" circuit for three inputs. (f) An "and" circuit for three inputs.

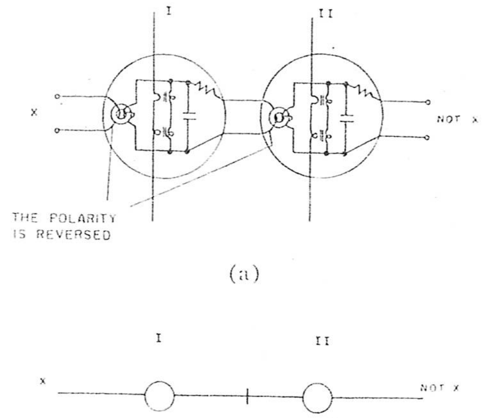

Fig. 9—(a) A "not" circuit. (b) The schematic diagram of the "not" circuit (a).

line being used per unit coupling intensity. Hence, a double line between circles will indicate that both parametrons are coupled at double coupling intensity (*cf.* Fig. 14). A short bar across any coupling line denotes complementation, that is, both parametrons are coupled with reversed polarity (Fig. 9), and otherwise it is understood that they are coupled in the same polarity.

If not specified, parametrons are supposed to be excited with the three-beat excitation. Accordingly, only parametrons belonging to different groups (I, II and III) can be coupled, and the information is transmitted along these lines always in the direction: I→II, II→III and III→I. Therefore, each coupling line has a definite direction of transmission, and to show this direction, usually the output lines from a parametron will come from the right side of the circle and go into the left side of another circle as input to it. As has been explained in Section III, there may be many parametrons which take some of their input signals from special parametrons called constant parametrons. These belong to a special triplet of parametrons, connected in a ring and always holding the digit "1," and serving as the phase reference. Since there may be a great many lines that come from these constant parametrons, these lines are usually omitted from the diagram, in order to avoid complication, and one "+" symbol is inscribed in the circle per unit constant input of positive polarity, and also one "−" symbol is inscribed per unit constant input of negative polarity. Accordingly, a circle with a "+" having two input lines corresponds to an "or" element, a circle with a "−" having two input lines corresponds to an "and" element, and a circle with "++" having three input lines corresponds to a 3-input "or" element, etc. It should be noted that the distinction between "0" and "1" in a parametron circuit is only possible by referring to the oscillation phase of these constant parametrons, since the phase is a relative concept.

The following figures show some simple examples of actual parametron circuits in schematic diagrams. The reader will not find it difficult to trace the functioning of these circuits. Fig. 10 shows a parametron flip-flop or a 1-bit memory circuit. Three parametrons, coupled in ring form, are required to store 1 bit of information. In Figs. 10(a) and 10(b) it is assumed that the signals in the set and reset inputs are both normally "0." The flip-flop will be set to "1" when a "1" signal is applied to the set input, and the flip-flop will be reset to "0" when a "1" signal is applied to the reset input. The functional difference between Figs. 10(a) and 10(b) consists in that, when both the set and the reset signal are applied simultaneously, the stored information will not change in the circuit of Fig. 10(a), but it will be reset to "0" in the circuit of Fig. 10(b).

Fig. 10(c) shows a flip-flop with a gate. As long as "0" is applied to the gate, the stored information does not change, but when "1" is applied to the gate, the signal from the input is transferred to the flip-flop.

Fig. 11 shows three stages of binary counting circuits connected in cascade, thus forming a scale-of-8 counter. Three flip-flops are included in this circuit to store a 3-bit count. In the quiescent state, in which "0" is applied to the input, the bits stored in each flip-flop do not change, but each time a "1" is applied to the input for a single clock period, the registered binary number is increased by 1 (mod 8). Figs. 12, 13 and 14 show respectively a binary full-adder circuit for three input signals, a parity check circuit for five input signals and a circuit for "x and (y or z)." These examples will show how majority operations can be made use of advantageously compared to the "and" and "or" operations. These circuits would have required many more parametrons if they were composed of "and" and "or" operations as in the usual diode networks. Flexibility of circuit design by use of a three- or five-input majority operation will be regarded as one of the characteristic features of parametron circuits.

The reason for the necessity of three subclock waves I, II and III, shown in Fig. 4, will be shown in Fig. 15. In Fig. 15, $P1$, $P2$, etc., indicate parametrons and $C1$, $C2$, etc., indicate coupling elements provided between parametrons. Each of the parametrons is supposed to be excited with either one of the two kinds of radio-frequency waves, I′, and II′, as shown in Fig. 16. These two waves are switched on and off alternately and will be called the two subclock exciting waves. If the coupling between two parametrons consists of a passive linear circuit, which is essentially a bilateral system, and if the two parametrons $P1$ and $P3$ in Fig. 15 are generating oscillations, voltages will be transmitted to $P2$ from both $P1$ and $P3$ with substantially the same intensity, and the phase-controlling action of $P2$ will become uncertain. Therefore, in order to use the two subclock exciting waves I′, and II′ in a parametron circuit, it is necessary to use unilateral coupling means. This may be accomplished by using a unilateral element, such as vacuum tubes and transistors, or by varying the coupling coefficient of the coupling elements as $K1$ and $K2$ in Fig. 16 by means of applying a gating signal to

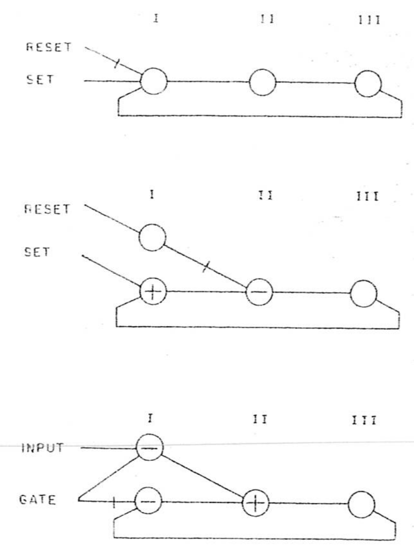

Fig. 10—Flip-flop circuits. (a) A flip-flop circuit. (b) A flip-flop circuit. (c) A flip-flop circuit with an input gate.

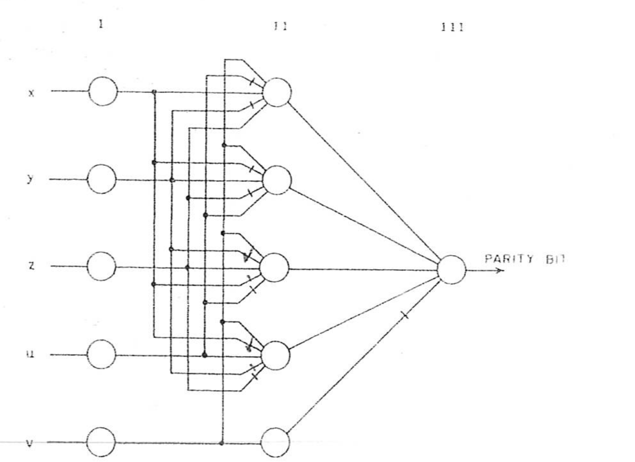

Fig. 13—A parity-check circuit for five input signals.

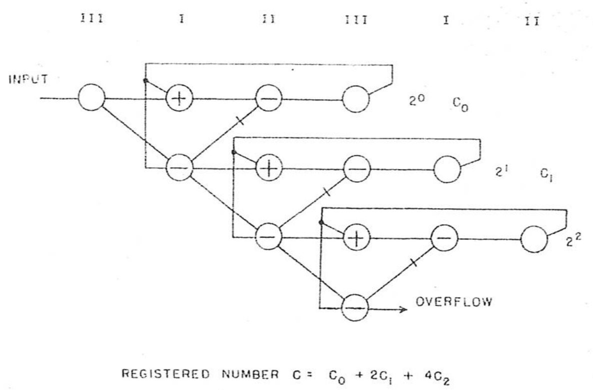

Fig. 11—Three stages of binary counters forming a scale of eight circuits.

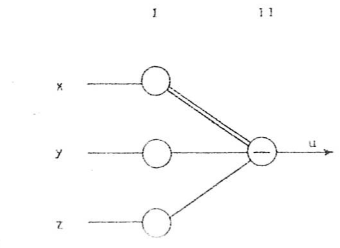

Fig. 14—A circuit for "x and (y or z)."

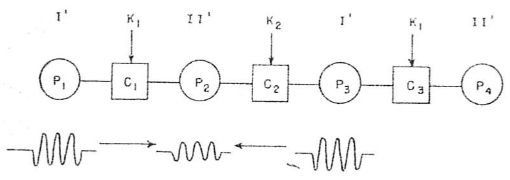

Fig. 15—Coupling system for two-subclock excitation.

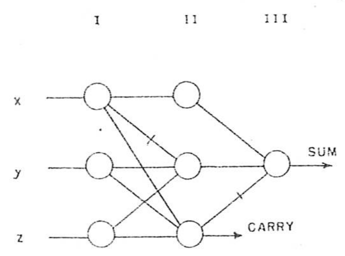

Fig. 12—A binary full-adder circuit.

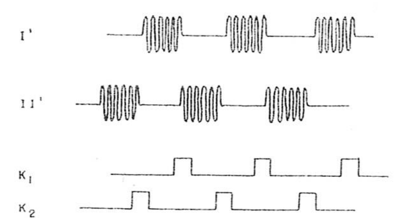

Fig. 16—Two-subclock excitation.

nonlinear elements, such as diodes and magnetic cores [27].

In the three-beat or the three-subclock exciting method, each of the parametrons will be excited once in every clock cycle at a definite time. In this respect the method may be called stationary excitation. On the other hand, we may think of a more general method, usually called "non-stationary excitation" or "gated excitation" in Japan, in which the excitation of parametrons is switched in accordance with gating signals [28]. Fig. 17 shows a selecting circuit using the "gated excitation." S_1, S_2, $\cdots$, S_n indicate binary phased information sources and P_1, P_2, $\cdots$, P_n are gating parametrons. Supposing that the exciting wave I is applied selectively to only one of the gating parametrons, say P_2, by controlling the excitation with a gating signal so as to produce oscillation only in P_2, the information from S_2 will be selectively transmitted to the parametron P, since the oscillation of P_2 is controlled by S_2 and the oscillation of P is controlled by that of P_2. For comparison, Fig. 18 shows a selecting circuit for one out of four channels using the three-subclock (stationary) ex-

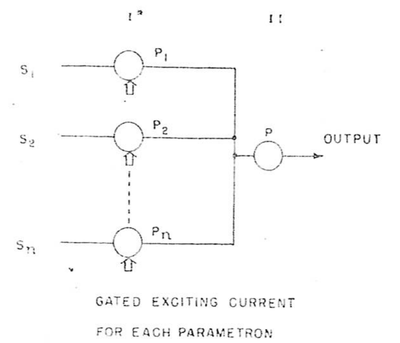

Fig. 17—Channel-selecting circuit using gated excitation.

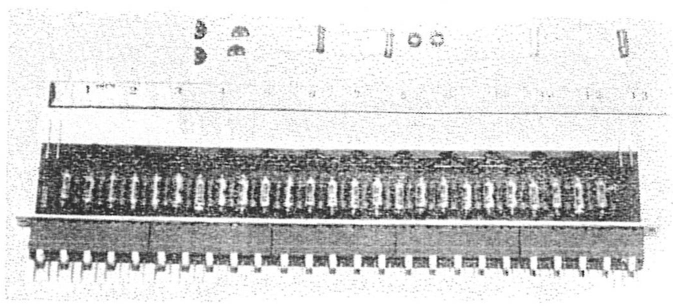

Fig. 19—A parametron unit (25 parametrons).

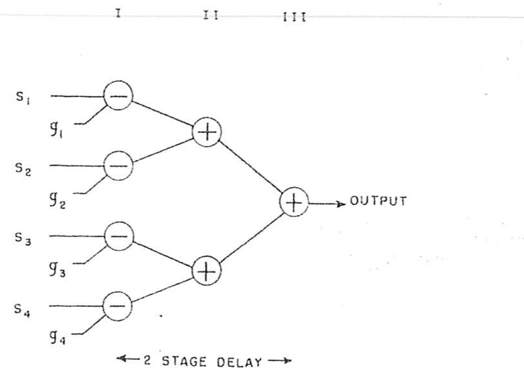

Fig. 18—Channel-selecting circuit using three-beat excitation.

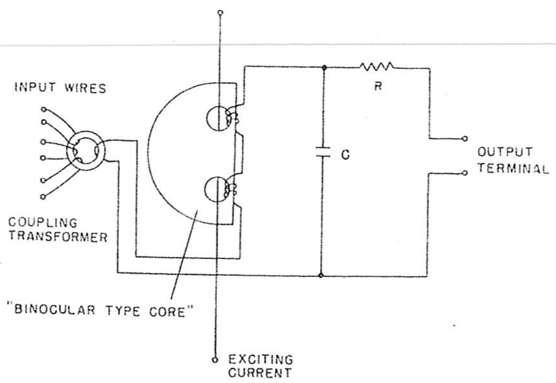

Fig. 20—The circuit of a parametron with a "binocular type core" and a series type coupling transformer.

citation. The channel selecting arrangement using gated excitation will generally reduce the access time and the number of parametrons at the expense of employing rather complicated exciting circuits.

V. Characteristic Features of Parametrons

Fig. 19 shows a commercial unit composed of 25 parametrons and the component parts. In this unit a ferrite disc with two small holes (known as a "binocular type core" [7]) is used instead of the two toroids in Fig. 3. The coupling transformer consists of a single-turn coil wound on a ferrite toroid and is connected in series to the resonant circuit as shown in Fig. 20. As the life of parametrons is considered to be practically permanent, the parametron units are usually not made in a "plug-in" style, but are directly wired into the logical networks.

As may be seen from Figs. 3, 7, and 8, the wiring of parametron circuits is done in an unusual way. A wire connected to the output terminal of a parametron in a preceding stage is passed through the coupling transformers of all the parametrons in the succeeding stage which are to receive the input signal from the parametron in the preceding stage. This has resulted in a remarkable simplification in the construction of complex logical networks, such as general purpose computers, since the whole system can be assembled from identical standardized units, and the units can be wired to form the specific machine using only wires and with a minimum number of soldering points. Table I shows the typical characteristics of commercial parametrons in Japan.

TABLE I

Characteristics of Commercial Parametron Units

	High Speed Type	Standard Type	Low Power Type
Exciting frequency $2f$	6 mc	2 mc	200 kc
Maximum clock frequency	140 kc	25 kc	2 kc
Exciting power per one parametron for continuous excitation	120 mw	30 mw	5 mw
DC bias	0.6 amp	0.6 amp	0.6 amp
Maximum number of inputs	3 or 5	3 or 5	3 or 5
Maximum number of output branching	12	15	15
Coupling coefficient*	−35 db	−40 db	−40 db

* *Note:* The coupling coefficient k is defined as the ratio:

$$k = \text{voltage of unit input measured at the resonant circuit} \Big/ \text{voltage of stationary oscillation}$$

For application to digital computers we are most concerned in the speed of operation, which is essentially determined by the clock frequency F_k. The upper limit of F_k is limited by the rates of building up and damping of parametric oscillation.

From (3) it follows that the oscillation builds up proportionally to $e^{\pi\Gamma ft}$ (*cf.* Appendix), and hence the maximum clock frequency will be proportional to the product Γf, which we call the "figure of merit" of the pararmetron, owing to its analogy to the figure of merit of vacuum tubes $g_m/2\pi C$.

The figure of merit of a parametron naturally depends on the frequency and amplitude of the exciting current, and the value for conventional parametrons in normal operation lies between 20kc and 1.5 mc. For reliable operation, the clock frequency should be chosen around $Pf/10$ if the coupling factor is $k=1/50$ (-34 db), and hence the upper limit for the clock frequency is about 150 kc for commercial parametron units.

In the past, most effort has been made to develop parametrons using variable inductance, but it is apparent that the same principle applies when the capacitor is the variable element. Parametrons using ceramic nonlinear capacitors (barium titanate) have been studied by Oshima and Kiyasu [8].

Studies of parametrons using the variable barrier capacitance of germanium and selenium diodes have also been made, and a parametric oscillation at $f=60$ mc has been realized [5], [9].

The high reliability and long life of parametrons, however, ...

and will limit their use in small-scale digital devices. At the present stage, parametrons seem to be unfavorably compared with vacuum tubes and transistors in speed of operation, but this point may be much improved by further development.

VI. Application

All the characteristics of parametrons just mentioned make them ideally suited to applications in large-scale digital devices, and particularly to general purpose digital computers. Soon after the invention of parametrons in 1954, a project was launched to construct general purpose computers using control and arithmetic units entirely composed of parametrons. At present, nearly half of the digital electronic computers built in Japan are parametron computers [11], [14]. Table II shows the characteristics of these computers.

TABLE II

The Characteristics of General Purpose Parametron Computers

Type (Date of Completion)	Place of Installation	The Number of Parametrons (Number System)	Exciting Frequency	Clock Frequency	Speed of Operation (for Fixed Point) Including Access		Main Memory	Power
					Addition	Multiplication		
FACOM 212 (March, 1959)	Fuji Elec. Co. Kawasaki	8000 (Decimal)	2 mc	10 kc	4 ms	15 ms	49 words Core Matrix	5 kw
HIPAC-1 (December, 1957)	Cint. Lab. Hitachi Elec. Co. Kokubunji, Tokyo	4400 (Binary)	2 mc	10 kc	10 ms	19 ms	1024 words Magnet Drum	6 kw
MUSASINO-1 (March, 1957)	Elec. Communication Lab. Musasino, Tokyo	5400 (Binary)	2 mc	6 kc	4 ms	20 ms	256 words Core Matrix	5 kw
NEAC-1101 (April, 1958)	Cent. Lab. Nippon Elec. Co. Kawasaki	3600 (Binary)	2 mc	20 kc	3.5 ms	8 ms	128 words Core Matrix	5 kw
PC-1 (March, 1958)	Department of Physics University of Tokyo Tokyo	4200 (Binary)	2 mc	15 kc	270 μs	3.4 ms	256 words Core Matrix	3 kw
PC-2* (August, 1959)	Department of Physics University of Tokyo Tokyo	9600 (Binary)	6 mc	100 kc	40 μs	340 μs	1024 words Core Matrix	10 kw
SENAC-1 (November, 1958)	Elec. Communication Lab. University of Tohoku Sendai	9600 (Binary)	2 mc	20 kc	2 ms	3 ms	160 words Magnetic Drum	15 kw

* *Note:* The construction of PC-2 will be completed in August 1959.

Parametrons are composed of capacitors, resistors and coils with ferromagnetic cores which are all stable and durable components. Unlike the more conventional switching circuits using magnetic amplifiers, parametrons require no diodes for their operation. These features guarantee for parametron circuits extremely high reliability and long life. In several digital computers now in operation in Japan, troubles with parametrons are extremely rare.

The necessity of a high-frequency power supply may be one of the inherent disadvantages of parametrons

In the core matrix memory of these parametron computers, an entirely new method, proposed by the author in 1955 [24]–[26], is employed both for reading and writing. Writing is effected by impressing on each memory core the superposition of two ac currents, supplied from parametrons and having frequencies of f and $f/2$. Reading is also effected with parametrons by amplifying and sensing the phase of the second harmonic component of frequency f which is generated in each memory core by impressing an ac current of frequency $f/2$ on it.

The new method is called "dual frequency memory system," and the following are considered to be characteristic features:

1) Memory cores are driven by output ac currents of parametrons.
2) Only two windings, X and Y, pass through each memory core.
3) Reading is nondestructive.

The details will be discussed in a separate paper to follow.

The application of parametrons to other digital devices has also been made in a number of laboratories. The Japan Overseas Telephone and Telegraph Company has constructed regenerative repeaters, telegraph code converters which convert Morse code to five-unit teleprinter code [6], and ARQ (automatic request) systems, which have all been in commercial use for some years.

The Japan Telegraph and Telephone Corporation has built a number of experimental common-control telephone switching systems, employing parametrons in control circuits [15]. The Fuji Electric Company and the Government Mechanical Laboratory have built experimental numerically controlled machine tools [13], in which parametrons are used for all numerical and control operations. Among other applications are automatic recording systems for a meson monitor used in cosmic-ray observation and multichannel pulse-height analyzers for nuclear research [4].

Appendix

Amplitude Limiting Mechanism of the Parametron

First, we shall derive the equation governing the oscillation in a parametric resonant circuit including a variable inductance $L(t)$ as shown in Fig. 21.

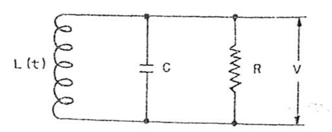

Fig. 21—A parametrically-excited resonant circuit.

The voltage V in the resonant circuit will be given by

$$V = \frac{d}{dt}(Li) \tag{4}$$

where i is the current passing through the inductance. From Kirchhoff's law, we obtain

$$i + \frac{V}{R} + \frac{d}{dt}(CV) = 0. \tag{5}$$

We shall assume that the inductance is varying as

$$L(t) = L_0(1 + 2\Gamma \sin 2\omega t). \tag{6}$$

Putting

$$I = \frac{L}{L_0} i = (1 + 2\Gamma \sin 2\omega t) i \tag{7}$$

$$\delta = \frac{1}{\omega CR} = \frac{1}{Q} \tag{8}$$

$$\frac{1}{CL_0} = \omega^2(1 + \alpha) \tag{9}$$

and assuming that Γ and α are much smaller than unity so that $(1+\alpha)(1+2\Gamma \sin 2\omega t)^{-1}$ may be replaced by $1+\alpha-2\Gamma \sin 2\omega t$, (6) will be rewritten as

$$\left[\frac{d^2}{dt^2} + \delta\omega \frac{d}{dt} + \omega^2(1 + \alpha - 2\Gamma \sin 2\omega t)\right] I = 0. \tag{10}$$

We may call δ the loss factor, α the detuning of the resonant circuit from the second-subharmonic frequency ω, Γ the modulation index of the inductance and L_0 the constant part of the inductance. As the difference between I defined by (7) and the actual current i is of the order of Γ, the results to be obtained from the following analysis of I may be regarded as substantially the same as that of the actual current i when Γ is small.

In case α and δ are constants, (10) represents a linear differential equation, well known as Mathieu's equation [16]–[18]. In practice, however, ferromagnetic cores are used in the inductance to effect the variation, and with increasing amplitude the nonlinear B-H curve will cause detuning of the resonant circuit and hysteresis loss will also increase. Consequently, the loss δ and the detuning α of (10) will generally be functions of the amplitude I^2 and (10) becomes a nonlinear differential equation.

Now, we shall assume the presence of nonlinearity of the form βI^2 as the detuning. Then (10) becomes

$$\left[\frac{d^2}{dt^2} + \delta\omega \frac{d}{dt} + \omega^2(1 + \alpha + \beta I^2 - 2\Gamma \sin 2\omega t)\right] I = 0. \tag{11}$$

Breaking down I into two sinusoidal components as

$$I = I_s \cdot \sin \omega t + I_c \cdot \cos \omega t, \tag{12}$$

(11) will be rewritten as

$$\begin{aligned}
&[2\dot{I}_s\omega + \delta\omega^2 I_s + \alpha\omega^2 I_c - \Gamma\omega^2 I_s + \tfrac{3}{4}\beta\omega^2(I_s^2 + I_c^2) I_c] \cos \omega t \\
&+ [-2\dot{I}_c\omega - \delta\omega^2 I_c + \alpha\omega^2 I_s - \Gamma\omega^2 I_c + \tfrac{3}{4}\beta\omega^2(I_s^2 + I_c^2) I_s] \sin \omega t \\
&+ [\ddot{I}_c + \delta\omega \dot{I}_c] \cos \omega t \\
&+ [\ddot{I}_s + \delta\omega \dot{I}_s] \sin \omega t \\
&+ [\Gamma\omega^2 I_s + \tfrac{1}{4}\beta\omega^2 I_c^3 - \tfrac{3}{4} I_s^2 I_c] \cos 3\omega t \\
&+ [-\Gamma\omega^2 I_c - \tfrac{1}{4}\beta\omega^2 I_s^3 + \tfrac{3}{4} I_s I_c^2] \sin 3\omega t = 0.
\end{aligned} \tag{13}$$

In order to obtain an approximate solution of the nonlinear differential equations (11) or (13), we shall assume that α, Γ and δ are much smaller than unity.

Then I_s and I_c in (13) will vary much more slowly than ω, and the third and fourth terms of (13) may be neglected since they are much smaller than ω^2. The third harmonic terms may also be neglected since they are off resonance and thus we will obtain the following approximate equations for I_s and I_c:

$$\frac{2}{\omega}\dot{I}_s = -\delta I_s + \Gamma I_s - (\alpha + \tfrac{3}{4}\beta(I_s^2 + I_c^2)) I_c$$

$$\frac{2}{\omega}\dot{I}_c = -\delta I_c - \Gamma I_s + (\alpha + \tfrac{3}{4}\beta(I_s^2 + I_c^2)) I_s. \tag{14}$$

Each term of (14) has the following intuitive meaning: the first term with δ represents the loss in the circuit; the second term with Γ indicates negative resistance effect for the sine component I_s and damping (positive resistance) effect for the cosine component I_c; and the third term with α and β represents detuning of the resonant circuit which is a function of the amplitude R,

$$R = \sqrt{I_s^2 + I_c^2}.$$

In case of no detuning, *i.e.*, $\alpha = 0$, and of small amplitude, the solution of (14) is given simply by

$$\begin{aligned} I_s &= I_{s0} \exp(\pi f(\Gamma - \delta)t) \\ I_c &= I_{c0} \exp(-\pi(\Gamma + \delta)t) \end{aligned} \tag{15}$$

where $\omega = 2\pi f$. Therefore, in case $\Gamma > \delta \geq 0$ holds, the sine component I_s will increase exponentially as described in Section II, while the cosine component I_c decreases exponentially.

The solution of a nonlinear differential equation such as (14) will be presented as integral curves or loci in the (I_s, I_c) plane and the behavior of these curves will be characterized by the singular points, *i.e.*, points in (I_s, I_c) plane where both $\dot{I}_s$ and $\dot{I}_c$ vanish (*cf.* [19], [20]).

The singular points of (14) will be obtained by placing $\dot{I}_s = \dot{I}_c = 0$ into (14) and the result may be classified into three cases 1, 2 and 3 depending on the magnitude of the parameters α, Γ and δ, as shown in Fig. 22. In Fig. 22, the abscissa represents $-\epsilon\alpha$ and the ordinate, δ, where $\epsilon = +1$ if $\beta > 0$ and $\epsilon = -1$ if $\beta < 0$. The characteristic curves which form the boundary lines of the three cases are two half-lines parallel to the α axis and a circle of radius Γ with its center at the origin. These three cases will be characterized by the following features:

Case 1

There are three singular points: One unstable saddle point at the origin $I_s = I_c = 0$, and two stable nodal or spiral points at

$$I_s = \pm\sqrt{\frac{2(\Gamma + \delta)}{3\Gamma|\beta|}(-\epsilon\alpha + \sqrt{\Gamma^2 - \delta^2})}$$

$$I_c = \pm\epsilon\sqrt{\frac{2(\Gamma - \delta)}{3\Gamma|\beta|}(-\epsilon\alpha + \sqrt{\Gamma^2 - \delta^2})}. \tag{16}$$

The integral curves of this case 1 have been shown in Fig. 5 for typical values $\alpha = 0$, $\delta = \Gamma/2$, $\beta < 0$. The existence of two stable states, the exponential build up of the small initial oscillation and all other characteristic features of parametrons described in Sections II and V will be explained by the behaviors of the integral curves of this Case 1.

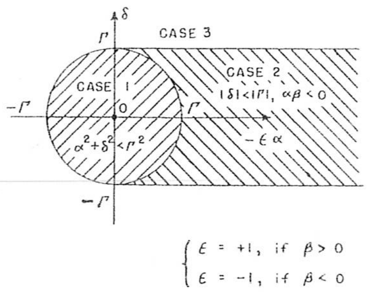

Fig. 22—Classification of the three cases of singular points in (α, δ) plane.

Case 2

There are five singular points: One stable nodal or spiral point at the origin $I_s = I_c = 0$, and two unstable saddle points at

$$I_s = \pm\sqrt{\frac{2(\Gamma + \delta)}{3|\beta|\Gamma}(-\epsilon\alpha - \sqrt{\Gamma^2 - \delta^2})}$$

$$I_c = \mp\epsilon\sqrt{\frac{2(\Gamma - \delta)}{3|\beta|\Gamma}(-\epsilon\alpha - \sqrt{\Gamma^2 - \delta^2})} \tag{17}$$

and two stable nodal or spiral points at

$$I_s = \pm\sqrt{\frac{2(\Gamma + \delta)}{3|\beta|\Gamma}(-\epsilon\alpha + \sqrt{\Gamma^2 - \delta^2})}$$

$$I_c = \pm\epsilon\sqrt{\frac{2(\Gamma - \delta)}{3|\beta|\Gamma}(-\epsilon\alpha + \sqrt{\Gamma^2 - \delta^2})}. \tag{18}$$

The integral curves of this Case 2 are shown in Fig. 23 for typical values $\delta = \Gamma/2$, $\alpha = 7\Gamma/4$, and $\beta < 0$. In Fig. 23, S, S' indicate the two unstable saddle points and A, A' indicate the two stable spiral points. The presence of the stable saddle point at the origin O indicates that the oscillation is not self-starting. If a suitable initiating voltage is applied to the circuit so as to place the point representing the initial oscillation either in the + region or in the − region of Fig. 23, stationary oscillation respectively represented by point A or A' will be produced. On the other hand, if the point representing the initial oscillation were within the O-region, even a voltage of very large amplitude would not initiate stationary oscillation. As there are three stable states respectively represented by O, A, A' in this Case 2, parametron elements corresponding to this case are usually called "tristable" or "ternary" parametrons, while those corresponding to Case 1 are called "bistable" or "binary" parametrons. In principle, a tristable parametron element may either represent a ternary digit by

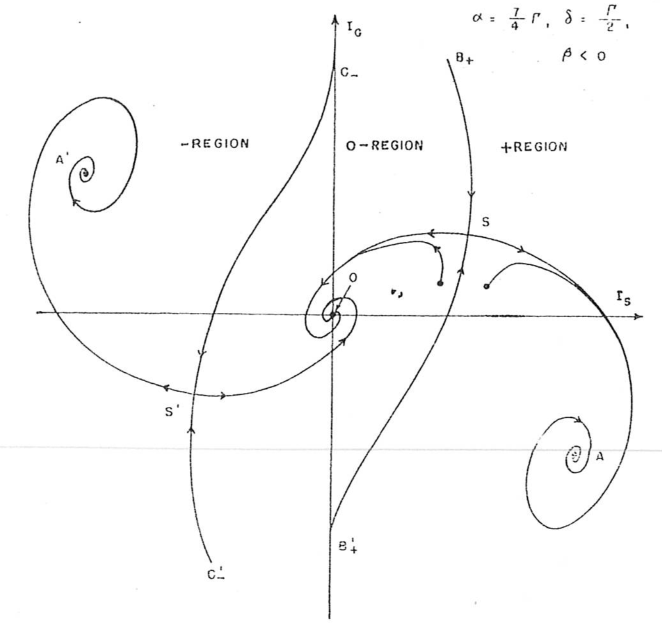

Fig. 23—Integral curves of a tristable parametron.

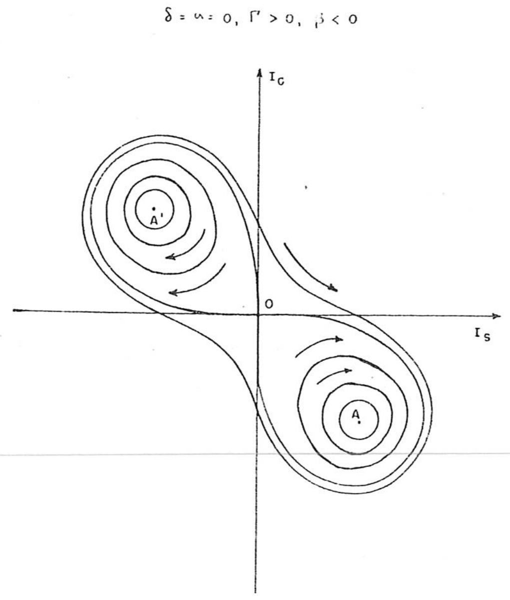

Fig. 24—Integral curves for a loss-free case.

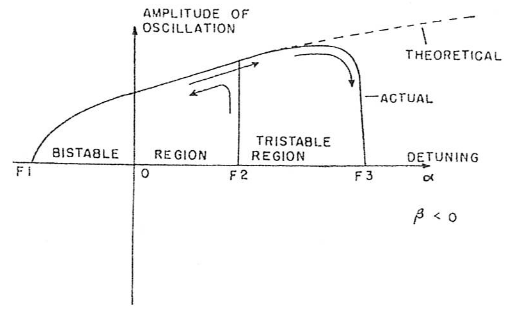

Fig. 25—Amplitude to detuning characteristic of a parametron.

the choice among the three stable states or a binary digit by the choice between the two states, namely, "no oscillation (O)" and "in oscillation (A and A′)."

Case 3

There is only one stable singular point at the origin. As the magnitude of the parameters α, Γ and δ are inappropriate, stationary oscillation is not produced in this case.

Now, the functions of the damping will be considered. If there were no damping in (14), *i.e.*, $\delta = 0$, the permissible types (*cf.* [19], [20]) of singular points will be unstable saddle points and elliptic points, the stability of the latter being neutral. Fig. 24 shows the integral curves for a case in which $\alpha = \delta = 0$, $\Gamma > 0$, $\beta < 0$. 0 indicates a saddle point at the origin and A, A' indicate two elliptic points. (For a point P on each curve, $AP \cdot A'P = \text{constant}$ is satisfied, and the curves are known as Cassini's ovals.) The point in the (I_s, I_c) plane, representing both the phase and the amplitude of the oscillation, will oscillate indefinitely around the points or point A and/or A', and a generally stationary state of oscillation with a definite amplitude and phase will never be reached. Further, if there were no damping, the oscillation in a parametron would never damp out, even if the parametric excitation were interrupted and it would be impossible to make use of the super-regenerative amplification explained in Section II.

Hence, we come to the following conclusion—damping is indispensable both for amplitude stabilization and interruption of parametric oscillation.

On the other hand, if the damping is too large, the building-up rate exp $(\pi f(\Gamma - \delta))$ of the sine component I_s, given by (15), will become so small as to reduce the speed of the superregenerative action. Therefore, there should exist an optimum value of the magnitude of the damping and experimental results show that the optimum value lies in the range $\Gamma/4 < \delta < \Gamma/2$.

Fig. 25 shows a typical example of the amplitude to detuning characteristic of an actual parametron element. In the figure, the abscissa represents the detuning α and the ordinate represents the amplitude of oscillation of a parametron element. In practice, the detuning α may be varied either by varying the tuning capacitance C in Fig. 3(a), or the tuning inductance L in Fig. 3(b), or by varying the frequency or the dc bias of the exciting current in the cases of both Figs. 3(a) and 3(b). The region $F1$ to $F2$ in Fig. 25 corresponds to the above mentioned case 1 and is called "bistable region" since it represents a bistable parametron. The region $F2$ to $F3$, corresponding to Case 2, is called "tristable region," since it represents a tristable parametron. When α is varied continuously a hysteresis jump will occur at the boundary $F2$ between the bistable and tristable region, as indicated by the arrows in Fig. 25. If we assume the presence of nonlinearity only in the detuning as in (11) and (14), the theoretical results indicate that the tristable region should extend indefinitely, as shown by the dotted line in Fig. 25 or by the two

half-lines in Fig. 22. Actually, there exists an upper limit $F3$ and this fact will be explained by introducing nonlinearity also in the damping, for example by replacing δ in (11) and (14) by $\delta + \theta I^2$. For bistable parametrons, however, the present analysis assuming the presence of nonlinearity only in the detuning is in good agreement with the experimental facts and generally it is considered satisfactory.

In regard to the nonlinearity of the detuning βI^2, one might think it were caused by saturation of the magnetic cores. If this were the case, β should be positive since the inductance would decrease and the detuning would increase with increasing amplitude. Experiments made on various ferrite and ferroelectric materials, however, show that β is always negative for these materials. On the other hand, it is observed that β is always positive for parametrons using barrier capacitance of semiconductor junctions.

Acknowledgment

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