Equivalence in Harmonics Between Cycloconverters and Bridge Converters

SHOTA MIYAIRI, HIROFUMI AKAGI, TADASHI FUKAO, AND MASAO FUJITA

***Abstract*—The similarity between a six-pulse three-phase cycloconverter operating in the circulating current-free mode and a six-pulse three-phase bridge converter is discussed analytically with regard to the rms value and the harmonics of the respective output voltages. A method is presented for calculating the rms voltage of a family of side-band frequencies which are generated in the output voltage of the cycloconverter. The results obtained from the analysis help to predict the amplitudes of subharmonics and beat frequencies with the cycloconverter and are essential for the design of the output filters. Analytical results are verified by comparison with those of experiment and digital computer simulation.**

INTRODUCTION

A GREAT deal of interest has been generated in the use of cycloconverters for high-power applications such as ball-mill drives and linear motor drives for high-speed ground transportation.

There are many possible circuit arrangements of the cycloconverter. The most basic cycloconverter which operates from a three-phase supply and is commutated naturally by the ac supply voltages is a six-pulse three-phase cycloconverter as shown in Fig. 1(a). This circuit consists of two conventional six-pulse three-phase bridge converters shown in Fig. 1(b), connected back-to-back with one another.

In the six-pulse three-phase bridge converter (hereinafter called the bridge converter), the firing angle of one phase is the same as that of other phases for steady-state dc output. On the other hand, the firing angle of each phase of the six-pulse three-phase cycloconverter (hereinafter called the cycloconverter) should be controlled so that the output voltage may be the desired ac voltage $E_0 \sin \omega_0 t$.

Although, from the viewpoint of the output waveform, there is a remarkable difference between the two converters, the behaviors of them are the same as each other during the period between the firing of one thyristor and the next. The bridge converter, therefore, may be considered a special type of the cycloconverter.

The fact described just above becomes more clear by comparing the frequency spectra of both output voltages as shown in Fig. 2(a) and Fig. 2(b). They are obtained by digital com-

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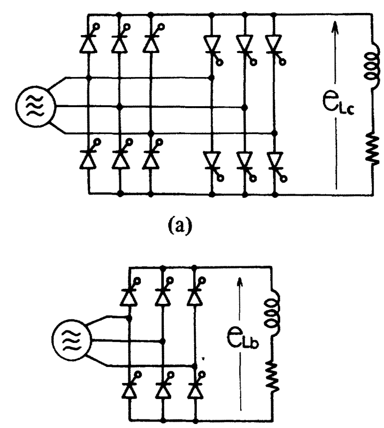

Fig. 1. Cycloconverter and bridge converter circuits. (a) Six-pulse three-phase cycloconverter. (b) Six-pulse three-phase bridge converter.

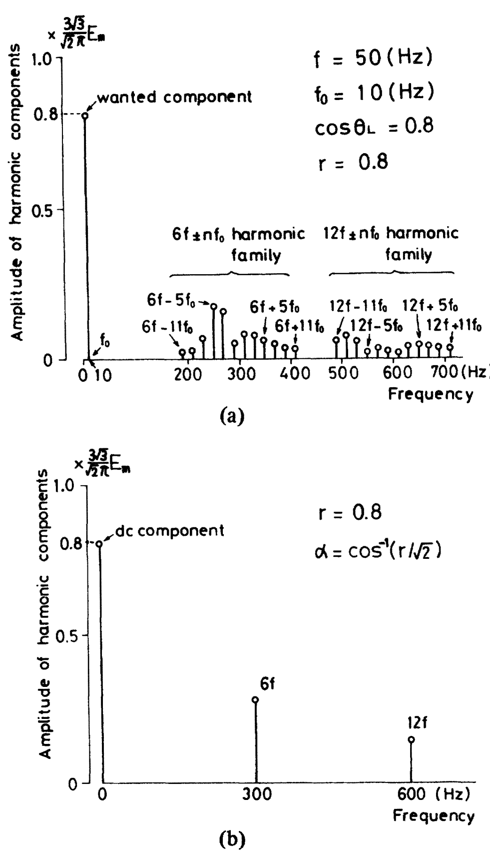

Fig. 2. Frequency spectra in the output voltage of the cycloconverter and the bridge converter. (a) Cycloconverter case. (b) Bridge converter case.

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puter simulation under the ideal condition. Fig. 2(a) shows the frequency specturm of the cycloconverter, where the frequencies of the harmonics f_{hc} are

$$f_{hc} = 6mf \pm nf_0, \qquad (m = 1, 2, 3, \cdots, n = 1, 3, 5, \cdots),$$

and f is the frequency of the supply and f_0 is the frequency of the desired output voltage. In this figure, $\cos\theta_L$ is the displacement factor of the load and r is the output voltage ratio defined in the later chapter. Fig. 2(b) shows that of the bridge converter, where the frequencies of the harmonics f_{hb} are

$$f_{hb} = 6mf, \qquad (m = 1, 2, 3, \cdots).$$

It should be noted that the harmonic components in the output voltage of the cycloconverter form a "family" of sideband frequencies around each harmonic components, which will be present in the output voltage of the bridge converter, and when the output frequency of the cycloconverter is equal to zero, i.e., $f_0 = 0$, f_{hc} coincides with f_{hb}.

A detailed analysis of the output frequency spectrum is given by Pelly [1]. The similarity between both converters, however, has not been clarified quantitatively yet. McMurray [2] has already developed a quantitative measure for the harmonic components in the output voltage of the cycloconverter, which is called "practical measures of total effect of each harmonic family" by him, considering the similarity of both converters. But, the results are obtained by numerical integration aided by digital computer and presented in graphical form. Explicit equations have not been derived yet. Moreover, this method is valid only for the case of $f_0 \ll f$.

In this paper, a general method of analysis that avoids the limitation mentioned above is developed. Some significant equations are derived which relate the output voltages of the bridge converter and the cycloconverter.

HARMONICS AND RMS VALUE OF THE OUTPUT VOLTAGE OF THE BRIDGE CONVERTER

In this section, the harmonics and the rms value of the output voltage of the bridge converter are treated, using the switching functions S_a, S_b, and S_c shown in Fig. 3.

Let e_a, e_b, and e_c be the instantaneous values of the supply:

$$\begin{aligned} e_a &= E_m \sin \omega t \\ e_b &= E_m \sin(\omega t - 2\pi/3) \\ e_c &= E_m \sin(\omega t - 4\pi/3) \end{aligned} \tag{1}$$

where E_m is the amplitude of the phase voltage and ω is equal to $2\pi f$.

Assuming the commutating overlap angle to be negligible, the instantaneous output voltage becomes as follows:

$$e_{Lb} = S_a e_a + S_b e_b + S_c e_c. \tag{2}$$

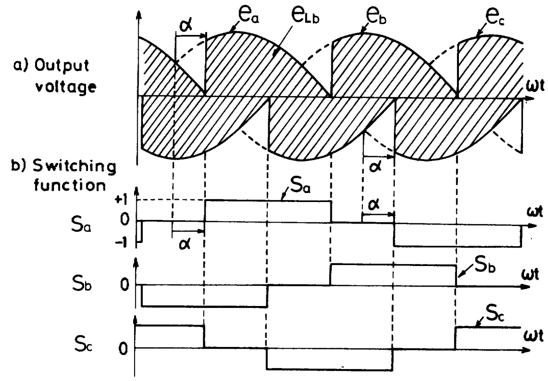

Fig. 3. Output voltage and switching functions of bridge converter.

According to Fourier analysis, S_a can be expanded to the following harmonic series:

$$S_a = \sum_{k=1,3,5,\cdots}^{\infty} (a_k \cos k\omega t + b_k \sin k\omega t) \tag{3}$$

where

$$a_k = \frac{2}{\pi}\int_{\pi/6+\alpha}^{5\pi/6+\alpha} \cos k\theta \, d\theta = \frac{2}{\pi}\cdot\frac{(-1)^{l+1}}{k}\sqrt{3}\sin k\alpha \tag{4}$$

$$b_k = \frac{2}{\pi}\int_{\pi/6+\alpha}^{5\pi/6+\alpha} \sin k\theta \, d\theta = \frac{2}{\pi}\cdot\frac{(-1)^{l}}{k}\sqrt{3}\cos k\alpha \tag{5}$$

$$k = 6l \pm 1, \qquad (l = 1, 2, 3, \cdots,)$$

Since S_b and S_c are delayed from S_a by $2\pi/3$ and $4\pi/3$, respectively, the Fourier series for S_b and S_c are obtained easily. Therefore, (2) becomes

$$\begin{aligned} e_{Lb} = (3E_m/2)\{&b_1 + (a_5 - a_7)\sin 6\omega t \\ &+ (b_7 - b_5)\cos 6\omega t + (a_{11} - a_{13})\sin 12\omega t \\ &+ (b_{13} - b_{11})\cos 12\omega t + \cdots\} \end{aligned} \tag{6}$$

Rearranging the above equation,

$$\begin{aligned} e_{Lb} = \frac{3\sqrt{3}}{\pi}E_m\{&\cos\alpha + (\tfrac{1}{5}\sin 5\alpha - \tfrac{1}{7}\sin 7\alpha)\sin 6\omega t \\ &+ (\tfrac{1}{5}\cos 5\alpha - \tfrac{1}{7}\cos 7\alpha)\cos 6\omega t + \cdots\}. \end{aligned} \tag{7}$$

The first term on the right side of (7) represents the mean voltage of e_{Lb}, that is,

$$H_0 = \frac{3\sqrt{3}}{\pi}E_m \cos\alpha. \tag{8}$$

The remaining terms on the right side of (7) represent harmonic components.

From (7), the rms voltage of $6f$ harmonic component is

$$\begin{aligned} H_{6f}{}^{b} &= \frac{3\sqrt{3}}{\pi}E_m\sqrt{\frac{1}{2}\left\{\left(\frac{1}{5}\sin 5\alpha - \frac{1}{7}\sin 7\alpha\right)^2 + \left(\frac{1}{5}\cos 5\alpha - \frac{1}{7}\cos 7\alpha\right)^2\right\}} \\ &= \frac{3\sqrt{3}}{\pi}E_m\sqrt{\frac{1}{2}\left(\frac{1}{5^2} + \frac{1}{7^2} - \frac{2}{5\cdot 7}\cos 2\alpha\right)}. \end{aligned} \tag{9}$$

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From an inspection of (7), we obtained the rms voltage of $6mf$ harmonic component as follows:

$$H_{6mf}{}^{b} = \frac{3\sqrt{3}}{\pi} E_m \sqrt{\frac{1}{2}\left\{\frac{1}{(6m-1)^2} + \frac{1}{(6m+1)^2} - \frac{2\cos 2\alpha}{(6m-1)(6m+1)}\right\}}. \tag{10}$$

Accordingly, the rms output voltage of the bridge converter is

$$E_{Lb} = \frac{3\sqrt{3}}{\pi} E_m \sqrt{\cos^2\alpha + \frac{1}{2}\sum_{m=1}^{\infty}\left\{\frac{1}{(6m-1)^2} + \frac{1}{(6m+1)^2} - \frac{2\cos 2\alpha}{(6m-1)(6m+1)}\right\}}$$
$$= \frac{3\sqrt{3}}{\pi} E_m \sqrt{\frac{1}{2}\left(1 + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{11^2} + \cdots\right) + \frac{1}{2}\left(1 - \frac{1}{5} + \frac{1}{7} - \frac{1}{11} + \cdots\right)\cos 2\alpha}$$

and further

$$E_{Lb} = E_m \sqrt{\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}\cos 2\alpha} \tag{11}$$

where

$$1 + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{11^2} + \cdots = \frac{\pi^2}{9}$$

$$1 - \frac{1}{5} + \frac{1}{7} - \frac{1}{11} + \cdots = \frac{\pi}{2\sqrt{3}}.$$

Equation (11) can be obtained in another way. That is, the rms value of the instantaneous output voltage e_{Lb} can be calculated according to the definition of the rms value, as follows:

$$E_{Lb} = \sqrt{\frac{3}{\pi}\int_{\pi/6+\alpha}^{\pi/2+\alpha} (e_a - e_b)^2 \, d\theta}. \tag{12}$$

By substituting $e_a - e_b$ with

$$e_a - e_b = \sqrt{3}E_m \cos(\theta - \pi/3),$$

(12) becomes

$$E_{Lb} = E_m \sqrt{\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}\cos 2\alpha}. \tag{13}$$

Evidently, (13) coincides with (11).

EQUIVALENCE IN HARMONICS AND RMS VALUE OF THE OUTPUT VOLTAGE BETWEEN BOTH CONVERTERS

rms Voltage of a Family of Sideband Frequencies

In this section, the rms voltage of the total harmonic components in the $6f \pm nf_0$ family, defined by

$$H_{6f} = \sqrt{\sum_{n=1,3,5,\cdots}^{\infty} (H_{6f-nf_0}{}^2 + H_{6f+nf_0}{}^2)}, \tag{14}$$

will be calculated.

The instantaneous voltage of the cycloconverter is expressed by using the switching functions S_a, S_b, and S_c shown in Fig. 4 and the mode functions M and M' shown in Fig. 5:

$$e_{Lc} = (S_a e_a + S_b e_b + S_c e_c)M + (S_a{}' e_a + S_b{}' e_b + S_c{}' e_c)M' \tag{15}$$

where $M = 1$ and $M' = 0$ when output current i_L is positive and $M = 0$ and $M' = -1$ when i_L is negative.

Let the fundamental output voltage, that is, wanted component of output voltage, be e_{Lf},

$$e_{Lf} = E_0 \sin \omega_0 t \tag{16}$$

where ω_0 is equal to $2\pi f_0$. Then the mode functions and the firing angles are given by

$$M(\omega_0 t) = 1, \quad M'(\omega_0 t) = 0$$
$$\alpha(\omega_0 t) = \cos^{-1}(r \cdot \sin \omega_0 t), \quad \text{when } i_L > 0, \tag{17}$$
$$M(\omega_0 t) = 0, \quad M'(\omega_0 t) = -1$$
$$\alpha'(\omega_0 t) = \cos^{-1}(-r \cdot \sin \omega_0 t), \quad \text{when } i_L < 0, \tag{18}$$

where r is output voltage ratio and defined as the ratio of the amplitude of the output voltage to the maximum possible mean dc voltage, that is,

$$r = E_0/(3\sqrt{3}E_m/\pi).$$

The firing angles $\alpha(\omega_0 t)$ and $\alpha'(\omega_0 t)$ are periodic functions of period $1/f_0$ and the switching functions are so-called phase-controlled waves as shown in Fig. 4. The instantaneous output voltage of the cycloconverter, e_{Lc} is derived by the same procedure as (6) obtained from (1):

$$e_{Lc} = \frac{3\sqrt{3}}{\pi} E_m [\{\cos\alpha + (\tfrac{1}{5}\sin 5\alpha - \tfrac{1}{7}\sin 7\alpha)\sin 6\omega t$$
$$+ (\tfrac{1}{5}\cos 5\alpha - \tfrac{1}{7}\cos 7\alpha)\cos 6\omega t + \cdots\}M(\omega_0 t)$$
$$+ \{\cos\alpha' + (\tfrac{1}{5}\sin 5\alpha' - \tfrac{1}{7}\sin 7\alpha')\sin 6\omega t$$
$$+ (\tfrac{1}{5}\cos 5\alpha' - \tfrac{1}{7}\cos 7\alpha')\cos 6\omega t + \cdots\}M'(\omega_0 t)]. \tag{19}$$

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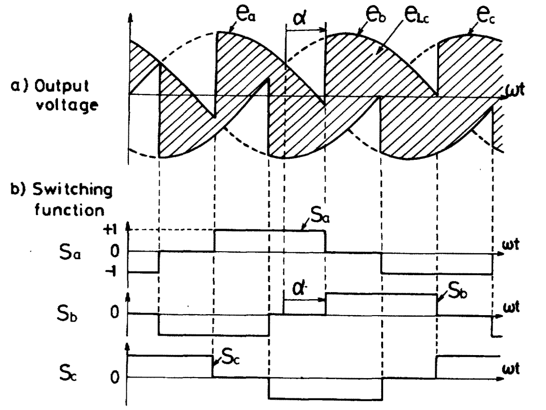

Fig. 4. Output voltage and switching functions of cycloconverter.

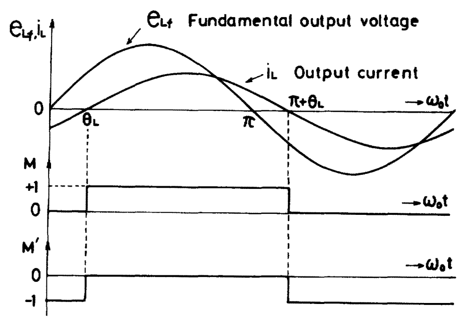

Fig. 5. Fundamental component of output voltage, output current, and mode functions.

Substituting (17) and (18) into (19), each harmonic component is obtained [3], and finally, from (14), the rms voltage of the total harmonic components in the $6f \pm nf_0$ family is

$$H_{6f} = \frac{3\sqrt{3}}{\pi} E_m \sqrt{\frac{1}{2}\left\{\frac{1}{5^2} + \frac{1}{7^2} - \frac{2}{5 \cdot 7}(r^2 - 1)\right\}}. \tag{20}$$

Generally, the rms voltages of the total harmonic components in the $6mf \pm nf_0$ harmonic families are

$$H_{6mf} = \frac{3\sqrt{3}}{\pi} E_m \sqrt{\frac{1}{2}\left\{\frac{1}{(6m-1)^2} + \frac{1}{(6m+1)^2} - \frac{2(r^2-1)}{(6m-1)(6m+1)}\right\}}. \tag{21}$$

And the rms value of the fundamental output voltage is

$$H_{f_0} = \frac{3\sqrt{3}}{\pi} E_m \frac{r}{\sqrt{2}}. \tag{22}$$

Detailed calculations are shown in the Appendix.

An inspection of (10) and (21) shows that the rms value of $6mf$ harmonic component in the output voltage of the bridge converter is equal to the total rms voltage of the $6mf \pm nf_0$ harmonic family of the cycloconverter, if α and r are related by

$$\cos\alpha = r/\sqrt{2}. \tag{23}$$

An inspection of (8) and (22) shows that the mean output voltage of the bridge converter H_0 is equal to the rms value of the fundamental output voltage of the cycloconverter H_{f0}, if $\cos\alpha = r/\sqrt{2}$.

Output rms Voltage of the Cycloconverter

In the general case where the input to output frequency ratio of the cycloconverter is an irrational number, the output waveform is not repetitive. Therefore, the rms value defined by (12) is no longer valid for the output quantities in the strict sense.

Let the rms output voltage of the cycloconverter be defined by

$$E_{Lc} = \sqrt{H_{f_0}^2 + \sum_{m=1}^{\infty} \sum_{n=1,3,5,\cdots}^{\infty} (H_{6mf-nf_0}^2 + H_{6mf+nf_0}^2)}$$
$$= \sqrt{H_{f_0}^2 + \sum_{m=1}^{\infty} H_{6mf}^2}. \tag{24}$$

In the case where the frequency ratio is a rational number, however, the output waveform is repetitive and its rms voltage can be obtained according to the definition of the rms value as follows,

$$E_{Lc} = \sqrt{\frac{1}{T}\int_0^T e_{Lc}^2 \, dt} \tag{25}$$

where T is one period of the output waveform of the cycloconverter. It should be noted that (24) does not coincide with (25), since several harmonic frequencies may take on the same values as one another.

For example, now let one of the harmonic frequencies in the $6f \pm nf_0$ harmonic family be $6f + n_6 f_0$, and let one in the $12f \pm nf_0$ harmonic family be $12f - n_{12} f_0$. Here, if n_6, n_{12}, f_0, and f are related by

$$(n_6 + n_{12}) f_0 / f = 6, \tag{26}$$

the following relationship exists:

$$f_{hc}' = 6f + n_6 f_0 = 12f - n_{12} f_0. \tag{27}$$

This result is that the amplitude of the f'_{hc} component is a vector sum of the amplitude of $6f + n_6 f_0$ component and that of $12f - n_{12} f_0$ component.

Case of the Irrational Ratio of f_0/f: Substituting (21) and (22) into (24), the output rms voltage of the cycloconverter is given by

$$E_{Lc} = \sqrt{H_{f_0}^2 + \sum_{m=1}^{\infty} H_{6mf}^2}$$
$$= E_m \sqrt{\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}(r^2 - 1)} \tag{28}$$

or

$$\frac{E_{Lc}}{V_l} = \sqrt{1 + \frac{3\sqrt{3}}{2\pi}(r^2 - 1)} \tag{28'}$$

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where V_l is the line voltage of the supply and is equal to $\sqrt{3} \cdot E_m/\sqrt{2}$.

Therefore, the rms output voltages of both converters are identical when $\cos\alpha = r/\sqrt{2}$, as shown in (13) and (28).

Equation (28′) indicates that E_{Lc} coincides with V_l when $r = 1$, that is, the maximum rms voltage of the cycloconverter is equal to the rms line voltage.

Case of the Rational Ratio of f_0/f: Assuming the following conditions:

supply frequency $f = 50$ Hz

output frequency $f_0 = 10$ Hz,

the following relation exists between the firing angle α_i and time t_i when the thyristor is turned on:

$$\alpha_i = \cos^{-1}(r \cdot \sin\omega_0 t_i), \qquad (i = 1, 2, \cdots, 15). \tag{29}$$

It is noted that (29) is valid during the period when i_L is positive. Therefore, the rms value of the output voltage is

$$E_{Lc} = \sqrt{\frac{1}{\pi}\int_{\theta_L}^{\pi+\theta_L} e_{Lc}^2 \, d(\omega_0 t)}$$
$$= E_m \sqrt{\frac{3}{2} + \frac{9\sqrt{3}}{4\pi} \cdot \frac{1}{15}\sum_{i=1}^{15} \cos 2\alpha_i}. \tag{30}$$

Here, α_i has a discrete value as expressed by (29). Equation (30) means that the output rms voltage depends on the output frequency. But, by assuming that α_i is also given by (17), an approximate expression of (30) can be obtained:

$$E_{Lc} \cong E_m \sqrt{\frac{3}{2} + \frac{9\sqrt{3}}{4\pi} \cdot \frac{1}{\pi}\int_{\theta_L}^{\pi+\theta_L} (2r^2 \sin^2\omega_0 t - 1)\, d(\omega_0 t)}$$
$$= E_m \sqrt{\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}(r^2 - 1)}. \tag{31}$$

Equation (31) coincides with (28).

VERIFICATION OF ANALYTICAL RESULTS BY EXPERIMENT AND DIGITAL COMPUTER SIMULATION

The experimental waveforms shown in Fig. 6 are the output voltage (upper) and the output current (lower) of the six-pulse three-phase cycloconverter operating from the following conditions:

supply frequency	$f = 50$ Hz,
supply line voltage	$V_l = 100$ V,
output frequency	$f_0 = 10$ Hz,
displacement factor of load	$\cos\theta_L = 0.8$ (lag),
output voltage ratio	$r = 0.8$.

Fig. 7 shows the experimental frequency spectrum of the output voltage shown in Fig. 6. It was measured by means of a wave-analyzer (HP 3581A) and a X–Y recorder. Fig. 7 agrees well with Fig. 2(a).

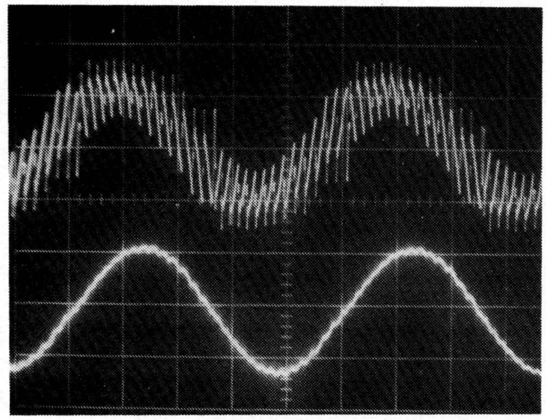

Fig. 6. Output voltage (upper) and output current (lower) waveforms of cycloconverter. (100 V/div, 10A/div, and time scale 20 ms/div).

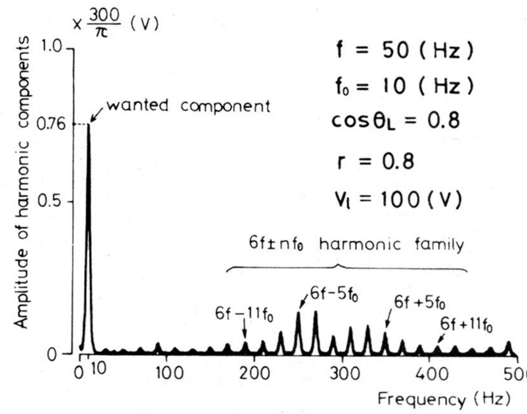

Fig. 7. Frequency spectrum in output voltage of cycloconverter (experiment).

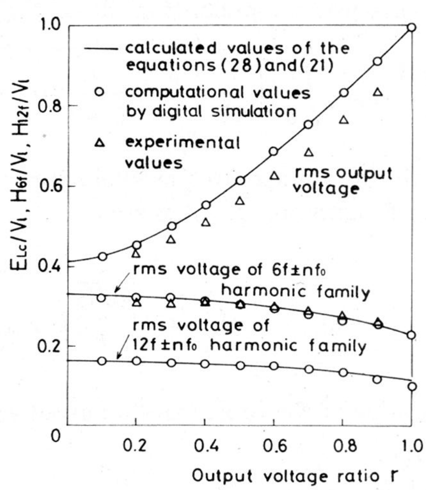

Fig. 8. Rms values of output voltage and those of both $6f \pm nf_0$ and $12f \pm nf_0$ components of cycloconverter.

The triangles in Fig. 8 are the experimental values of the rms output voltage and the rms voltage of the $6f \pm nf_0$ harmonic family, as the output voltage ratio ranges from 0.2–0.9 under the above conditions. The points in Fig. 8 indicate the values obtained by digital computer simulation under the same conditions. And the harmonic components are considered up to $6f \pm 11f_0$ and $12f \pm 11f_0$, respectively. The solid curves in Fig. 8 show the analytical values obtained by using (28) and (21).

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It is shown that the theory developed in this paper is in good agreement with the results obtained by digital computer simulation. Moreover, this implies that the rms output voltage of the cycloconverter is given by (28) approximately, even when the ratio of input to output frequency is a rational number. The experimental values are somewhat different to the analytical and computational values of the rms output voltage. This difference is due to the fact that the analytical and computational values are obtained under the assumption of neglecting the forward voltage drop of thyristors and the source inductances.

CONCLUSION

In this paper, similarities in the harmonics and the rms voltages between the bridge converter and the cycloconverter are discussed analytically, and obtained the following results.

1) The output voltages of the cycloconverter and the bridge converter are equal in rms value, if the firing angle α for the bridge converter and the output voltage ratio r for the cycloconverter are related by

$$\cos\alpha = r/\sqrt{2}. \tag{32}$$

Equation (32) implies that the mean voltage at dc terminals of the bridge converter is equal to the rms value of the fundamental output voltage of the cycloconverter.

2) The rms values for $6mf \pm nf_0$ ($m = 1, 2, 3, \cdots, n = 1, 3, 5, \cdots$) harmonic families in the output voltage of the cycloconverter coincide with the rms voltage of $6mf$ harmonic components in the bridge converter.

Therefore, the $6mf$ harmonic components of the bridge converter can be considered so-called "degenarate states" of the harmonics in $6mf \pm nf_0$ harmonic families of the cycloconverter.

APPENDIX

DERIVATION OF (20), (21), AND (22) FROM (19)

First, consider the $6f \pm nf_0$ harmonic family. The instantaneous voltage of this harmonic family is

$$e_{Lc}{}^{6f} = \frac{3\sqrt{3}}{\pi} E_m \{A(\omega_0 t)\sin 6\omega t + B(\omega_0 t)\cos 6\omega t\} \tag{33}$$

where

$$A(\omega_0 t) = \{\tfrac{1}{5}\sin 5\alpha(\omega_0 t) - \tfrac{1}{7}\sin 7\alpha(\omega_0 t)\}M(\omega_0 t) + \{\tfrac{1}{5}\sin 5\alpha'(\omega_0 t) - \tfrac{1}{7}\sin 7\alpha'(\omega_0 t)\}M'(\omega_0 t) \tag{34}$$

and

$$B(\omega_0 t) = \{\tfrac{1}{5}\cos 5\alpha(\omega_0 t) - \tfrac{1}{7}\cos 7\alpha(\omega_0 t)\}M(\omega_0 t) + \{\tfrac{1}{5}\cos 5\alpha'(\omega_0 t) - \tfrac{1}{7}\cos 7\alpha'(\omega_0 t)\}M'(\omega_0 t). \tag{35}$$

By Fourier analysis, (34) and (35) are

$$A(\omega_0 t) = A_{S1}\sin\omega_0 t + A_{C1}\cos\omega_0 t + A_{S3}\sin 3\omega_0 t + A_{C3}\cos 3\omega_0 t + \cdots$$
$$B(\omega_0 t) = B_{S1}\sin\omega_0 t + B_{C1}\cos\omega_0 t + B_{S3}\sin 3\omega_0 t + B_{C3}\cos 3\omega_0 t + \cdots \tag{36}$$

where

$$A_{Sn} = \frac{1}{\pi}\int_0^{2\pi} A(\omega_0 t)\sin n\omega_0 t\, d(\omega_0 t)$$
$$A_{Cn} = \frac{1}{\pi}\int_0^{2\pi} A(\omega_0 t)\cos n\omega_0 t\, d(\omega_0 t)$$
$$B_{Sn} = \frac{1}{\pi}\int_0^{2\pi} B(\omega_0 t)\sin n\omega_0 t\, d(\omega_0 t)$$
$$B_{Cn} = \frac{1}{\pi}\int_0^{2\pi} B(\omega_0 t)\cos n\omega_0 t\, d(\omega_0 t),$$
$$(n = 1, 3, 5, \cdots).$$

Substituting (34) and (35) into (33), the following expression is obtained:

$$e_{Lc}{}^{6f} = \frac{3\sqrt{3}}{\pi} E_m\{\tfrac{1}{2}(A_{C1} - B_{S1})\sin(6\omega - \omega_0)t + \tfrac{1}{2}(A_{S1} + B_{C1})\cos(6\omega - \omega_0)t + \tfrac{1}{2}(A_{C1} + B_{S1})\sin(6\omega + \omega_0)t + \tfrac{1}{2}(B_{C1} - A_{S1})\cos(6\omega + \omega_0)t + \cdots\}. \tag{37}$$

Therefore, the rms voltages of the $6f - f_0$ and $6f + f_0$ harmonics are, respectively,

$$H_{6f-f_0} = \frac{3\sqrt{3}}{\pi}\cdot\frac{E_m}{2}\sqrt{\tfrac{1}{2}\{(A_{C1} - B_{S1})^2 + (B_{C1} + A_{S1})^2\}}$$
$$H_{6f+f_0} = \frac{3\sqrt{3}}{\pi}\cdot\frac{E_m}{2}\sqrt{\tfrac{1}{2}\{(A_{C1} + B_{S1})^2 + (B_{C1} - A_{S1})^2\}}.$$

The rms value of $e_{Lc}{}^{6f}$ is given by

$$H_{6f} = \sqrt{\sum_{n=1,3,5,\cdots}^{\infty}(H_{6f-nf_0}{}^2 + H_{6f+nf_0}{}^2)} = \frac{3\sqrt{3}}{\pi}\cdot\frac{E_m}{2}\sqrt{A_{S1}{}^2 + A_{C1}{}^2 + A_{S3}{}^2 + A_{C3}{}^2 + \cdots + B_{S1}{}^2 + B_{C1}{}^2 + B_{S3}{}^2 + B_{C3}{}^2 + \cdots}. \tag{38}$$

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Here, it should be noted, that by the definition of the rms value, the following relations exist:

$$\frac{1}{2\pi}\int_{\theta_L}^{2\pi+\theta_L} A^2(\omega_0 t)\, d(\omega_0 t) = \tfrac{1}{2}(A_{S1}{}^2 + A_{C1}{}^2 + A_{S3}{}^2 + A_{C3}{}^2 + \cdots) \tag{39}$$

and

$$\frac{1}{2\pi}\int_{\theta_L}^{2\pi+\theta_L} B^2(\omega_0 t)\, d(\omega_0 t) = \tfrac{1}{2}(B_{S1}{}^2 + B_{C1}{}^2 + B_{S3}{}^2 + B_{C3}{}^2 + \cdots). \tag{40}$$

Since there is a following relationship between M and M', as shown by (17) and (18),

$$M(\omega_0 t) \cdot M'(\omega_0 t) = 0$$

then, from (34) and (35), the following is obtained:

$$A^2(\omega_0 t) + B^2(\omega_0 t) = \left\{\frac{1}{5^2} + \frac{1}{7^2} - \frac{2}{5 \cdot 7}\cos 2\alpha(\omega_0 t)\right\} M^2(\omega_0 t) + \left\{\frac{1}{5^2} + \frac{1}{7^2} - \frac{2}{5 \cdot 7}\cos 2\alpha'(\omega_0 t)\right\} M'^2(\omega_0 t).$$

Combining (39), (40), and the above equation gives

$$\frac{1}{2\pi}\int_{\theta_L}^{2\pi+\theta_L} \{A^2(\omega_0 t) + B^2(\omega_0 t)\}\, d(\omega_0 t) = \frac{1}{2\pi}\left[\int_{\theta_L}^{\pi+\theta_L}\left(\frac{1}{5^2} + \frac{1}{7^2} - \frac{2}{5 \cdot 7}\cos 2\alpha\right) d(\omega_0 t) + \int_{\pi+\theta_L}^{2\pi+\theta_L}\left(\frac{1}{5^2} + \frac{1}{7^2} - \frac{2}{5 \cdot 7}\cos 2\alpha'\right) d(\omega_0 t)\right]. \tag{41}$$

Here, from (17) and (18),

$$\cos 2\alpha(\omega_0 t) = \cos 2\alpha'(\omega_0 t) = 2r^2 \sin^2 \omega_0 t - 1$$

then,

$$\frac{1}{2\pi}\int_{\theta_L}^{2\pi+\theta_L} \cos 2\alpha\, d(\omega_0 t) = r^2 - 1.$$

Therefore, (41) becomes

$$\frac{1}{2\pi}\int_{\theta_L}^{2\pi+\theta_L} \{A^2(\omega_0 t) + B^2(\omega_0 t)\}\, d(\omega_0 t) = \frac{1}{5^2} + \frac{1}{7^2} - \frac{2}{5 \cdot 7}(r^2 - 1). \tag{42}$$

From (39), (40), and (42),

$$\frac{1}{2}(A_{S1}{}^2 + A_{C1}{}^2 + A_{S3}{}^2 + A_{C3}{}^2 + \cdots + B_{S1}{}^2 + B_{C1}{}^2 + \cdots) = \frac{1}{5^2} + \frac{1}{7^2} - \frac{2}{5 \cdot 7}(r^2 - 1). \tag{43}$$

Combining (38) and (43) gives

$$H_{6f} = \frac{3\sqrt{3}}{\pi} E_m \sqrt{\frac{1}{2}\left\{\frac{1}{5^2} + \frac{1}{7^2} - \frac{2}{5 \cdot 7}(r^2 - 1)\right\}} \tag{44}$$

and generally

$$H_{6mf} = \frac{3\sqrt{3}}{\pi} E_m \sqrt{\frac{1}{2}\left\{\frac{1}{(6m-1)^2} + \frac{1}{(6m+1)^2} - \frac{2(r^2-1)}{(6m-1)(6m+1)}\right\}}. \tag{45}$$

From (19), the rms voltage of the fundamental output component is

$$H_{f_0} = \frac{3\sqrt{3}}{\pi} \cdot E_m \sqrt{\frac{1}{2\pi}\int_{\theta_L}^{2\pi+\theta_L} (M\cos\alpha + M'\cos\alpha')^2\, d(\omega_0 t)} = \frac{3\sqrt{3}}{\pi} E_m \cdot \frac{r}{\sqrt{2}}. \tag{46}$$

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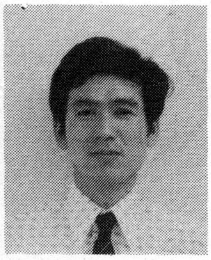

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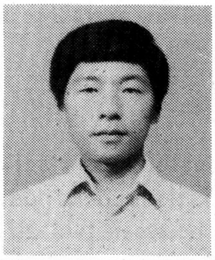

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