A New Approach to the Definition of Power Components in Three-Phase Systems Under Nonsinusoidal Conditions

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***Abstract*—The problem of defining power components in three-phase systems under nonsinusoidal situations is discussed. A new approach is proposed, which employs the description of three-phase systems in terms of Park vectors. Finally, it is shown how this method fits with other proposed methods that can be now regarded in terms of this more general theory.**

I. Introduction

THE PROBLEM of defining nonactive powers under nonsinusoidal conditions has been already discussed by several authors who defined power components both in the time domain (Fryze [1], Kusters and Moore [2], Page [3]) and in the frequency domain (Budeanu [4], Shepherd and Zakikhani [5], Sharon [6], Czarnecki [7], [8]).

The definitions proposed by these authors try to attain, as their final goal, the maximization of the power factor and largely originate from mere mathematical considerations, while only Czarnecki's theory attempts to give physical meaning to its definitions.

Furthermore, most given definitions deal with single-phase systems. Three-phase systems have been taken into consideration recently and the new proposed theories appear to be a mere extension of the given definitions [9]–[12].

Although these definitions seem quite attractive, they are concerned with the decomposition of currents into orthogonal components, rather than with power definitions. Nonactive powers are then defined as the product of the rms value of voltage by the rms value of current components.

However, this procedure leads to the definition of quantities that are intrinsically apparent powers: The formal properties of powers in electrical systems are not satisfied, since they have no sign, they cannot be algebraically added and they do not satisfy the conservation principle. Moreover, their measurement is very often difficult.

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These drawbacks can be overcome if the Park transformation is employed in describing three-phase systems: in fact, this mathematical approach represents a powerful, synthetic, and universal way to represent the behavior of three-phase systems in any possible working condition (unsymmetrical, unbalanced, nonsinusoidal, etc.) [13], [14].

The present paper will briefly illustrate the Park theory and will show how this theory can be applied to describe three-phase systems. General validity power definitions in three-phase systems will be derived, proposing a more general approach than Akagi's [15].

Moreover, if three-wire three-phase systems are considered, their representation in terms of Park vectors leads to a formalism to some extent similar to that employed to represent single-phase systems. It will be shown how currents decomposition proposed by other theories can be extended by employing the Park transformation.

II. Park Transformation

Park transformation is widely employed to study the behavior of rotating electrical machines in transient conditions [16]. However, it can be considered a more general and powerful tool to study the behavior of three-phase systems.

The Park transformation applied to the signals $y_a(t)$, $y_b(t)$, and $y_c(t)$ (voltages and currents) of a three-phase system leads to the Park components $y_d(t)$, $y_q(t)$, and $y_o(t)$ defined as

$$\begin{bmatrix} y_d \\ y_q \\ y_o \end{bmatrix} = [T] \cdot \begin{bmatrix} y_a \\ y_b \\ y_c \end{bmatrix} \tag{1}$$

where $[T]$ is the orthogonal matrix

$$[T] = \begin{bmatrix} \sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{6}} \\ 0 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} \end{bmatrix}.$$

In the d–q plane, it is then possible to define the Park vector as the complex quantity

$$y = y_d + jy_q \tag{2}$$

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thus changing the original quantities $y_a(t)$, $y_b(t)$, and $y_c(t)$ into the Park vector $\boldsymbol{y}(t)$ plus the zero-sequence component $y_o(t)$, both depending on time t.[1]

If the complex operator $\overline{\alpha} = e^{j2\pi/3}$ is introduced, the Park vector can be also obtained as:

$$\boldsymbol{y} = \sqrt{2/3} \cdot (y_a + \overline{\alpha} \cdot y_b + \overline{\alpha}^2 \cdot y_c). \quad (3)$$

The inverse Park transformation comes immediately from (1):

$$\begin{bmatrix} y_a \\ y_b \\ y_c \end{bmatrix} = [T]^{-1} \cdot \begin{bmatrix} y_d \\ y_q \\ y_o \end{bmatrix} = [T]' \cdot \begin{bmatrix} y_d \\ y_q \\ y_o \end{bmatrix} \quad (4)$$

since matrix $[T]$ is orthogonal and so $[T]^{-1} = [T]'$.

III. Three-Phase Systems Representation

Starting from the above considerations, it is possible to represent any three-phase system in terms of the Park vectors of line voltages and currents, plus their zero-sequence components.

If the zero-sequence components are not present,[2] the three-phase system can be represented only by these two Park vectors in the d–q plane, with a formalism similar to the phasorial one employed in the representation of single-phase sinusoidal systems.

Moreover, since the Park vector is invariant with respect to an additive term (that is the same for the three phases), it represents the "pure" three-phase component of the system.[3] The phase angle between the voltage and current Park vectors depends on the load nature and on the arbitrary phase order assigned to the phases.[4]

In periodical conditions (with period T) it is also possible to define a three-phase rms value [14] as

$$Y = \sqrt{\frac{1}{T} \cdot \int_T \boldsymbol{y}(t) \cdot \boldsymbol{y}^*(t) \cdot dt} \quad (5)$$

where $\boldsymbol{y}^*(t)$ is the conjugate vector of $\boldsymbol{y}(t)$.

[1]The proposed transformation is a particular case (sometimes referred to as the Clarke transformation) of a more general transformation that considers the d–q axes rotating. Employing this general formulation is possible, but does not represent any advantage for the purpose of the present paper.

[2]This is a very important case although it is a particular case. In fact, it will be shown in the following paragraphs that, as far as power concepts are involved, it is possible to neglect the zero-sequence components, provided that at least one of them is not present: This is the typical case of three-wire three-phase systems, in which the zero-sequence component of current is nil.

[3]When three-wire three-phase systems are concerned, this allows us to determine the Park vector of voltages starting from the line-to-neutral voltages referred to any arbitrary artificial neutral point. The Park vector is independent from the chosen neutral point. This leads to the determination of the Park vector of voltages starting from the line-to-line voltages as:

$$\boldsymbol{v} = \frac{\sqrt{2}}{3} [v_{ab} + \overline{\alpha} \cdot v_{bc} + \overline{\alpha}^2 \cdot v_{ca}] \cdot e^{-j(\pi/6)}.$$

[4]It must be noted that, while phase shift between voltage and current phasors represents a time shift between sinewaves and only depends on the load nature, phase shift between voltage and current Park vectors represents a spatial phase shift in the d–q plane and depends on the load nature and on the assigned phase order.

If the zero-sequence component is nil, it follows [14]:

$$Y = \sqrt{Y_a^2 + Y_b^2 + Y_c^2}. \quad (6)$$

The rms value of the Park vector is so defined to be the "pure" three-phase rms value. This definition can be kept valid even if in the presence of the zero-sequence component; (6) is no longer valid in this case.

Some further interesting considerations, in order to perceive the wide generality of the Park representation, come from the study of the relationship between the Park vectors and the symmetrical components in sinusoidal conditions, and from the analysis of the Park vectors in the frequency domain.

A. Symmetrical Components

In sinusoidal conditions the phasorial formalism can be employed as well as the decomposition into symmetrical components.

Taking into account the matrix $[\overline{S}]$ defined as:

$$[\overline{S}] = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & \overline{\alpha} & \overline{\alpha}^2 \\ 1 & \overline{\alpha}^2 & \overline{\alpha} \\ 1 & 1 & 1 \end{bmatrix}$$

the phasors of the symmetrical components are obtained:

$$\begin{bmatrix} \overline{Y}_1 \\ \overline{Y}_2 \\ \overline{Y}_0 \end{bmatrix} = [\overline{S}] \cdot \begin{bmatrix} \overline{Y}_a \\ \overline{Y}_b \\ \overline{Y}_c \end{bmatrix}. \quad (7)$$

If the Park transformation (1) is now applied, the Park vector is obtained [14]:

$$\boldsymbol{y}(t) = (\overline{Y}_1 e^{j\omega t} + \overline{Y}_2^* e^{-j\omega t}) \quad (8)$$

and the zero-sequence component:

$$y_o(t) = \sqrt{2} \cdot \mathrm{Re}\, [\overline{Y}_0 e^{j\omega t}]. \quad (9)$$

Equation (8) shows the relation between the Park vector and the symmetrical components in sinusoidal conditions with angular frequency ω. In particular, if the three-phase system is balanced and with positive phase order, the Park vector can be obtained from the phasor of the positive symmetrical component $\overline{Y}_1$ only. On the other hand, if the three-phase system is balanced, but with negative phase order, the Park vector can be obtained from the conjugate of the phasor of the negative symmetrical component $\overline{Y}_2$ only.

B. Frequency-Domain Analysis

If the three-phase system is supposed to be periodical with period $T = 2\pi/\omega$, the Park components are periodical too, due to the Park transformation linearity. It is then possible to determine their Fourier series components.

In particular, the Park vector can be decomposed in the complex Fourier series [14]:

$$\boldsymbol{y}(t) = \sum_{-\infty}^{+\infty} \boldsymbol{Y}_k e^{jk\omega t} \quad (10)$$

whose terms are Park vectors with constant amplitude Y_k and with rotating speed proportional to the index k in positive and negative direction (except the term with index $k = 0$, corresponding to the DC component).

Each harmonic frequency $n\omega$, n a positive integer, is so described by two vectors ($k = n$, $k = -n$) with different amplitude, and rotating at the same speed, but with opposite directions.[5]

If (8) is reminded, for any specified harmonic n, the harmonic component of the Park vector at angular frequency $k\omega$, $k = n > 0$, is associated with the positive symmetrical component at angular frequency $n\omega$, while the harmonic component at angular frequency $k\omega$, $k = -n < 0$, is associated with the negative symmetrical component at the same angular frequency. It results:

$$\boldsymbol{Y}_k = \overline{Y}_{1n}, \qquad \text{for } k > 0;$$

$$\boldsymbol{Y}_k = \overline{Y}^*_{2n}, \qquad \text{for } k < 0.$$

The Park theory considers these pairs of terms as if they were different harmonic components, thus achieving a significant formal unity in the representation of distorted and nonsymmetrical systems.

At last, the rms value of the Park vector can be obtained from the rms values of its harmonic components in the same way as for quantities in single-phase systems. Since

$$\begin{aligned} y^2(t) &= \boldsymbol{y}(t) \cdot \boldsymbol{y}^*(t) \\ &= \sum_{-\infty}^{+\infty}{}_k\, Y_k^2 + \sum_{-\infty}^{+\infty}{}_{k \neq h}\, \boldsymbol{Y}_k \cdot \boldsymbol{Y}_h^*\, e^{j(k-h)\omega t} \end{aligned}$$

averaging this equation over the period T, the desired rms value is obtained:

$$Y = \sqrt{\sum_{-\infty}^{+\infty}{}_k\, Y_k^2}. \tag{11}$$

IV. Power Definitions

Since the Park matrix $[T]$ is orthogonal, the instantaneous power $p(t)$ can be obtained starting from the original phase voltages and currents:

$$p(t) = [v_a \quad v_b \quad v_c] \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

as well as starting from the Park components:

$$p(t) = [v_d \quad v_q \quad v_o] \cdot \begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix}. \tag{12}$$

Introducing the Park vectors of voltages $\boldsymbol{v}(t)$ and currents $\boldsymbol{i}(t)$, the Park instantaneous complex power can be defined as

$$\boldsymbol{a}_p(t) = \boldsymbol{v}(t) \cdot \boldsymbol{i}^*(t). \tag{13}$$

Expanding (13), the Park real power can be defined as

$$p_p(t) = \text{Re}\,[\boldsymbol{a}_p(t)] = v_d i_d + v_q i_q \tag{14}$$

as well as the Park imaginary power can be defined as [15]

$$q_p(t) = \text{Im}\,[\boldsymbol{a}_p(t)] = v_q i_d - v_d i_q \tag{15}$$

so that the Park complex power can be rewritten as

$$\boldsymbol{a}_p(t) = p_q(t) + j q_q(t). \tag{16}$$

The physical meaning of the real power can be immediately understood if the zero-sequence power

$$p_o(t) = v_o i_o \tag{17}$$

is introduced; from (12) and (14), it follows that

$$p(t) = p_p(t) + p_o(t). \tag{18}$$

The Park real power represents the instantaneous power in a three-phase system if either voltage or current zero-sequence component is not present.

The Park imaginary power is a characteristic quantity of the three-phase systems. It does not involve instantaneous three-phase power [15]; it arises when the instantaneous values of the Park vectors of voltages and currents are spatially shifted [13]. This happens when the ratio between the instantaneous values of the line voltages and the line currents are not the same for the three phases.

The above definitions are discussed in [15] too, mainly to represent the behavior of static compensators. However, the above-defined quantities represent a more powerful approach to describe three-phase systems under nonsinusoidal, unbalanced conditions. The average values of these quantities, in periodical conditions, can be usefully defined and related to quantities defined by other theories, as it will be shown later.

The average value of (16) can be evaluated:

$$\boldsymbol{A}_p = P_p + jQ_p = \frac{1}{T}\int_T \boldsymbol{v}(t) \cdot \boldsymbol{i}^*(t) \cdot dt \tag{19}$$

where P_p is the active power if $p_o(t) = 0$.

The defined quantities $\boldsymbol{a}_p(t)$, $p_p(t)$, $q_p(t)$, $p_o(t)$, as well as their average values, can be actually qualified as powers, and not apparent powers, since they satisfy the following properties[6]:

- they are algebraic quantities, whose sign depends on the reference direction assumed for voltages and currents;
- they satisfy the conservation principle: it can be proven that the algebraic sum of each of the defined

[5]The terms with opposite indexes are independent since they are the terms of the Fourier series of the complex function $\boldsymbol{y}$ (on the contrary, in case of real functions they were conjugate).

[6]It is worthwhile to point out that if the general form of the Park transformation is employed, the above-defined quantities are not affected by the position and the speed of the d-q axes [13], thus supporting the formal correctness of all given definitions.

powers concerned with each three-phase element of an isolated network is null.[7]

Moreover, these quantities have a further interesting property: They can be measured by means of simple linear combinations (with constant coefficients) of products of line voltages and currents as clearly indicated by (1), (14), (15), and (17).

In sinusoidal and balanced conditions, the real and imaginary Park powers are directly related to the active P and reactive Q powers. In fact, if a three-phase balanced system has symmetrical positive sequence three-phase voltages and currents, (3) leads to the following Park vectors:

$$\boldsymbol{v}(t) = \sqrt{3} V_l e^{j\omega t}$$
$$\boldsymbol{i}(t) = \sqrt{3} I_l e^{j(\omega t - \varphi)}$$

where V_l and I_l are the rms values of the line voltages and currents and φ is the phase displacement between them.

Applying (14) and (15), it is possible to obtain both real and imaginary Park powers as

$$p_p(t) = 3V_l I_l \cos \varphi = P_p = P;$$
$$q_p(t) = 3V_l I_l \sin \varphi = Q_p = Q \qquad (20)$$

which are constant and equal to the active and reactive power, respectively.

If symmetrical negative three-phase voltages and currents are considered, the Park vectors are

$$\boldsymbol{v}(t) = \sqrt{3} V_l e^{-j\omega t}$$
$$\boldsymbol{i}(t) = \sqrt{3} I_l e^{-j(\omega t - \varphi)}.$$

The real and imaginary Park powers can now be obtained as

$$p_p(t) = 3V_l I_l \cos \varphi = P_p = P;$$
$$q_p(t) = -3V_l I_l \sin \varphi = Q_p = -Q. \qquad (21)$$

It can be noted that they are still constant, but that the imaginary power is now equal to the opposite of the reactive power: This is in agreement with (8).

In nonsinusoidal, unbalanced periodical conditions, the instantaneous complex power can be obtained from the Fourier series components of the voltage and current Park vectors. Applying (10) to them yields to

$$\boldsymbol{a}_p(t) = \sum_{-\infty}^{+\infty}{}_k \boldsymbol{V}_k \cdot \boldsymbol{I}_k^* + \sum_{-\infty}^{+\infty}{}_{k \neq h} \boldsymbol{V}_k \cdot \boldsymbol{I}_h^* e^{j(k-h)\omega t}. \qquad (22)$$

[7]If the line voltages of an isolated three-phase network satisfy the Kirchhoff voltage law phase by phase, then the Park voltage components will satisfy the same law too, due to the linearity of the Park transformation. For the same reason, if the line currents of the same network separately satisfy the Kirchhoff current law, the Park current components will satisfy the same law too. The Tellegen theorem affirms that the sum of the products of any pairs of voltage and current components extended to all elements of the network is null. It follows that the sums of each defined power are null, since they are sums of such products.

Averaging (22) on the period, it can be obtained:

$$\boldsymbol{A}_p = P_p + jQ_p = \sum_{-\infty}^{+\infty}{}_k \boldsymbol{V}_k \cdot \boldsymbol{I}_k^*. \qquad (23)$$

From (23), the average value of the Park real power can be expressed by

$$P_p = \mathrm{Re}\left[\sum_{-\infty}^{+\infty}{}_k \boldsymbol{V}_k \cdot \boldsymbol{I}_k^*\right] = \sum_{-\infty}^{+\infty}{}_k P_k$$

where P_k are the active powers associated with each harmonic component and with each symmetrical component.

From (23), the average value of the Park imaginary power can be likewise expressed by

$$Q_p = \mathrm{Im}\left[\sum_{-\infty}^{+\infty}{}_k \boldsymbol{V}_k \cdot \boldsymbol{I}_k^*\right] = \sum_{-\infty}^{+\infty}{}_k Q_k.$$

Equations (20) and (21) lead to

$$Q_p = \sum_{1}^{+\infty}{}_n Q_{1n} - \sum_{1}^{+\infty}{}_n Q_{2n} + Q_{po} \qquad (24)$$

having considered the symmetrical components for each harmonic $n = |k|$. Q_{1n} is the reactive power associated with the positive sequence of harmonic n and Q_{2n} is the reactive power associated with the negative sequence of harmonic n.

It must be noted that the average value of the Park imaginary power is given by the sum of the reactive powers Q_{1n}, while the reactive powers Q_{2n} are subtracted.[8]

Moreover, the presence of the component with index $k = 0$ in Q_p can be noted because the DC component of the current in the Park vectors representation can be spatially shifted with respect to the voltage.

A. Apparent Power and Power Factor

Starting from the given definitions of three-phase rms values of voltages and currents, the following "pure" three-phase apparent power can be defined:

$$S = V \cdot I \qquad (25)$$

that is in agreement with the definition given in [17].

Unlike all other defined powers, this apparent power is not an algebraic quantity and does not satisfy the conservation principle.

[8]Although (24) is formally similar to the Budeanu definition of the reactive power Q_B, the two quantities are different. In fact, the Budeanu reactive power in a three-wire three-phase system can be written as

$$Q_B = \sum_{1}^{\infty}{}_n Q_{1n} + \sum_{1}^{\infty}{}_n Q_{2n}.$$

It can be noted that the reactive powers associated with the negative sequence are added, while in (24) they are subtracted. Moreover, the Park imaginary power has a physical meaning completely different from that of the Budeanu reactive power. In particular, it 1) is a characteristic quantity of the three-phase systems and it does not exist in single phase systems; 2) is not related to instantaneous power oscillations; 3) gives useful information on how to improve the power factor (as it will be shown in the following sections); 4) is defined in the time domain, while Budeanu reactive power is not; 5) can be easily measured, while Budeanu reactive power cannot.

The three-phase power factor can be defined as

$$\lambda = \frac{P_p}{S} \tag{26}$$

Taking into account the amplitude A_p of the average complex power A_p defined by (19):

$$A_p = \sqrt{P_p^2 + Q_p^2} \tag{27}$$

it can be proven [14] that it is always: $S \geq A_p$.

The quantity

$$D_p^2 = S^2 - A_p^2 \tag{28}$$

can be defined[9] so that the apparent power can be rewritten as

$$S^2 = P_p^2 + Q_p^2 + D_p^2. \tag{29}$$

B. Numerical Examples

To exemplify the proposed theory, the Park transformation was applied to the three-phase circuit of Fig. 1 for sinusoidal and nonsinusoidal supply voltages and for symmetrical and nonsymmetrical loads.

Fig. 2 shows the polar diagram of the voltage and current Park vectors for sinusoidal supply voltages ($e_a(t) = \sin(\omega t)$; $e_b(t) = \sin(\omega t - 2\pi/3)$; $e_c = \sin(\omega t + 2\pi/3)$; $\omega = 2\pi f$; $f = 50$ Hz); and for a symmetrical load ($R_a = R_b = R_c = 0.3\ \Omega$; $L_a = L_b = L_c = 0.001$ H); as it can be noted, the two diagrams are circular, since the Park vectors have constant amplitude in this case.

Fig. 3 shows the diagrams of the real and imaginary Park powers, in the same case as Fig. 2: they are constant and equal to the active and reactive powers, as expected. It results: $P_p = 2.385$ W, $Q_p = 2.497$ VAr, $V = 1.225$ V, $I = 2.819$ A, $S = 3.453$ VA, $A_p = 3.453$ VA, $\lambda = 0.691$.

Fig. 4 shows the polar diagram of the voltage and current Park vectors for the same sinusoidal supply voltages as those of Fig. 2, but for a nonsymmetrical load ($R_a = R_c = 0.3\ \Omega$, $R_b = 0.5\ \Omega$; $L_a = L_c = 0.001$ H, $L_b = 0.003$ H); as it can be noted, the diagram of the current vector is no longer circular, since the currents are not balanced.

Fig. 5 shows the diagrams of the real and imaginary Park powers, that are not constant, in the same case as that of Fig. 4. It results: $P_p = 1.641$ W, $Q_p = 2.006$ VAr, $V = 1.225$ V, $I = 2.238$ A; $S = 2.742$ V · A, $A_p = 2.592$ V · A, $\lambda = 0.598$.

Fig. 6 shows the polar diagram of the voltage and current Park vectors for nonsinusoidal supply voltages ($e_a = |\sin(\omega t)|$; $e_b = |\sin(\omega t - 2\pi/3)|$; $e_c = |\sin(\omega t + 2\pi/3)|$, $\omega = 2\pi f$, $f = 50$ Hz) and for a symmetrical load ($R_a = R_b = R_c = 0.3\ \Omega$; $L_a = L_b = L_c = 0.001$ H).

Fig. 7 shows the diagrams of the real and imaginary Park powers, that are not constant, in the same case as that of Fig. 6. It results: $P_p = 0.169$ W, $Q_p = -0.342$ VAr, $V = 0.531$ V, $I = 0.751$ A; $S = 0.399$ VA, $A_p = 0.381$ VA, $\lambda = 0.424$. It can be noted that $V^2 \neq V_a^2 + V_b^2 + V_c^2$, since the zero-sequence component of the three-phase voltages is not null.

[9] Once again, it is worth while to note that, although it is apparently formally identical, D_p is not the Budeanu distortion power, since Q_p is not the Budeanu reactive power.

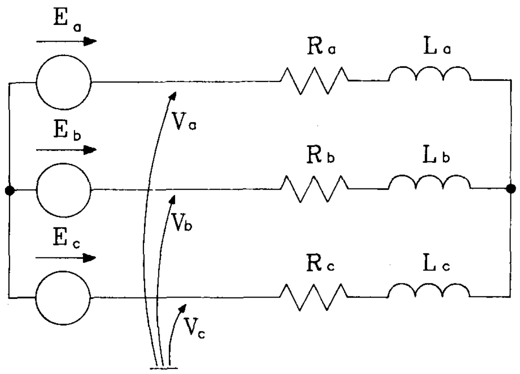

Fig. 1. Three-phase circuit employed to exemplify Park theory.

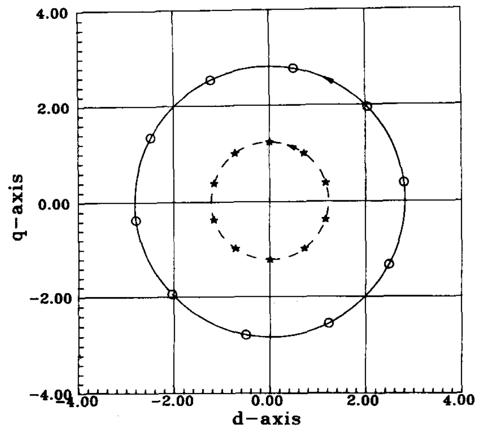

Fig. 2. Polar diagram of voltage (☆) and current (○) Park vectors in case of balanced sinusoidal supply voltage and symmetrical load.

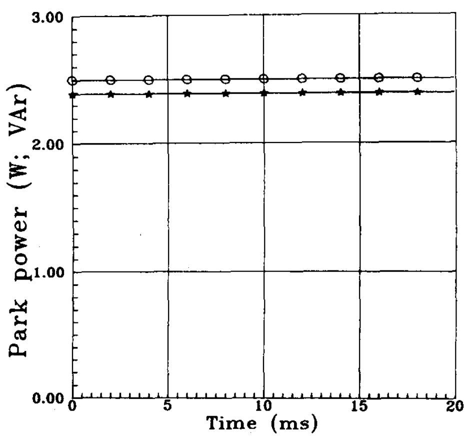

Fig. 3. Diagram of real (☆) and imaginary (○) power in the same case as that of Fig. 2.

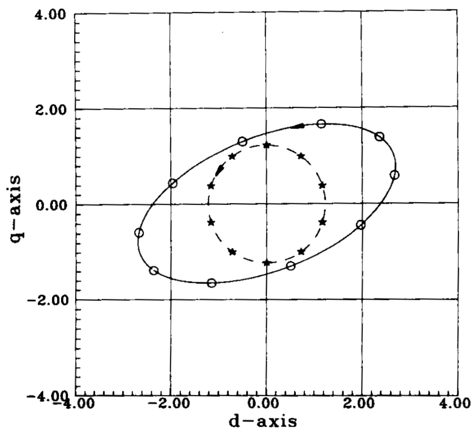

Fig. 4. Polar diagram of voltage (☆) and current (○) Park vectors in case of balanced sinusoidal supply voltage and nonsymmetrical load.

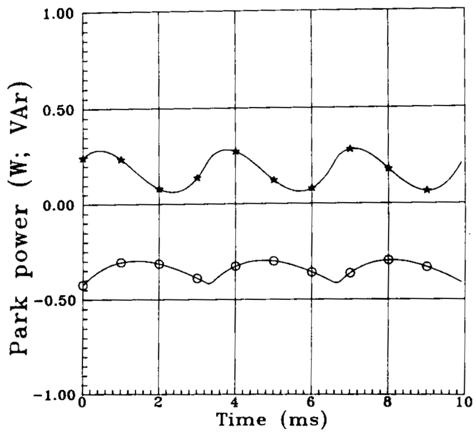

Fig. 7. Diagram of real (☆) and imaginary (○) power in the same case as that of Fig. 6.

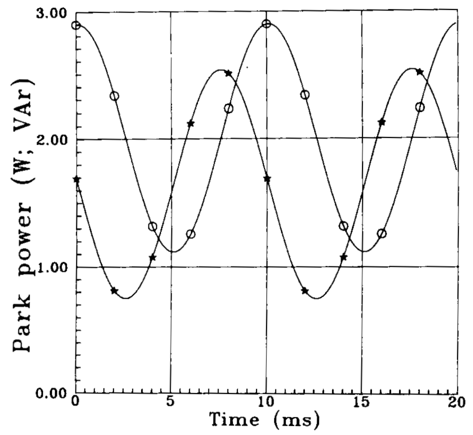

Fig. 5. Diagram of real (☆) and imaginary (○) power in the same case as that of Fig. 4.

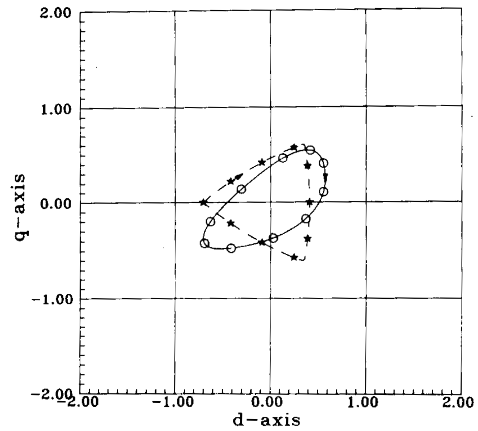

Fig. 8. Polar diagram of voltage (☆) and current (○) Park vectors in case of nonsinusoidal supply voltage and nonsymmetrical load.

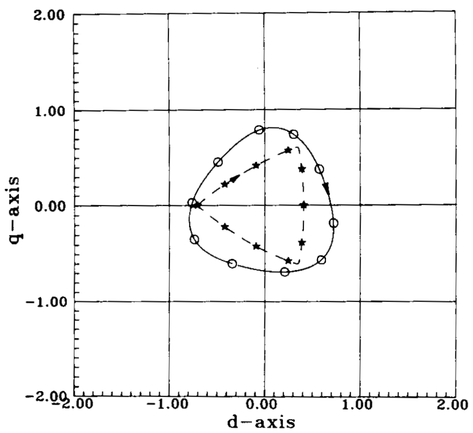

Fig. 6. Polar diagram of voltage (☆) and current (○) Park vectors in case of nonsinusoidal supply voltage and symmetrical load.

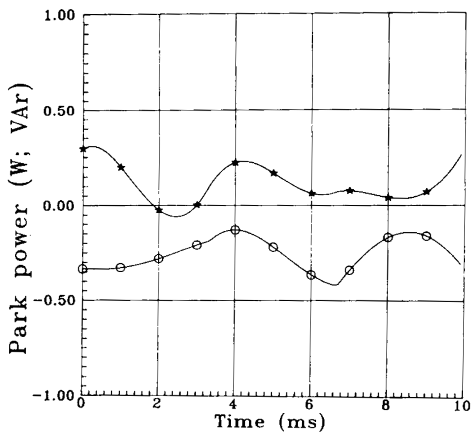

Fig. 9. Diagram of real (☆) and imaginary (○) power in the same case as that of Fig. 8.

Fig. 8 shows the polar diagram of the voltage and current Park vectors for the same nonsinusoidal supply voltages as those of Fig. 6, but for a nonsymmetrical load ($R_a = R_c = 0.3\ \Omega$, $R_b = 0.5\ \Omega$; $L_a = L_c = 0.001$ H, $L_b = 0.003$ H).

Fig. 9 shows the diagrams of the real and imaginary

Park powers in the same case as that of Fig. 8. It results: $P_p = 0.110$ W, $Q_p = -0.254$ VAr, $V = 0.531$ V, $I = 0.583$ A; $S = 0.310$ VA, $A_p = 0.277$ VA, $\lambda = 0.355$.

V. Power Compensation

When assessing the validity of the definitions of powers in nonsinusoidal conditions, their effectiveness in allowing correct economical evaluation of energy consumption as well as in allowing correct compensation of nonactive powers must be taken into account besides their correctness from a mere formal point of view.

It is not worthwhile, in this context, to examine economical problems, since they deeply involve extratechnical interests.

As far as power compensation is concerned, total compensation is achieved if $\lambda = 1$ is obtained. To attain this goal, the total compensation of the instantaneous values of $q_p(t)$ is necessary, although not sufficient [15].

Useful indications on how to accomplish its compensation are suggested by its physical meaning. Since it does not involve instantaneous three-phase power, it can be completely compensated without employing energy storage elements[10] [15]. This is a fundamental result if imaginary power compensation is to be attained with static converters: Their need for reactive elements is only due to the finite commutation frequency of the switching elements, not to system requirements. Moreover, the compensation of $q_p(t)$ leads to the maximum compensation without employing energy storage elements [15].

However, if the optimal compensation ($\lambda = 1$) is the goal, the instantaneous power $p_p(t)$ must be considered. An indication on how to operate is given by the extension of Fryze's time-domain decomposition [1] in terms of Park vectors.

Moreover, the frequency-domain decomposition [7]–[9] gives useful indications on the source of distortion and on the relationship between the average values of the real and imaginary Park powers and the current components. Indications on the effect of the compensation of the average value of the imaginary Park power can be obtained as well.

The following sections will discuss both time-domain and frequency-domain decompositions in terms of the Park vectors.

A. Time-Domain Decomposition

The Park vector approach leads to a straightforward extension of Fryze's [1] and Kusters and Moore's [2] theories to the three-phase systems. The current Park vector of a three-phase system with null zero-sequence component can be decomposed into an active component (that is associated to the active power) and a residual one.

[10]This does not exclude the possibility of employing such elements in imaginary power compensation, but only affirms that energy storage elements are not necessary. This is just the contrary of what happens in reactive power compensation in single-phase systems.

The active current is defined by

$$\boldsymbol{i}_a(t) = \frac{P_p}{V^2} \cdot \boldsymbol{v}(t) \tag{30}$$

where P_p is the average value of the real power and V the rms value of the voltage Park vector.

The residual current is obtained by the difference:

$$\boldsymbol{i}_x(t) = \boldsymbol{i}(t) - \boldsymbol{i}_a(t). \tag{31}$$

The functions $\boldsymbol{i}_a(t)$ and $\boldsymbol{i}_x(t)$ are orthogonal, since it results, for their scalar product:

$$\int_T \mathrm{Re}\,[\boldsymbol{i}_a(t) \cdot \boldsymbol{i}_x^*(t)] \cdot dt = 0.$$

This leads to the following equation for the rms values:

$$I^2 = I_a^2 + I_x^2 \tag{32}$$

and consequently to

$$P_p = \frac{1}{T}\int_T \boldsymbol{v}(t) \cdot \boldsymbol{i}_a^*(t) \cdot dt = VI_a. \tag{33}$$

This means that if $\boldsymbol{i}_x$ is completely compensated, so that $\boldsymbol{i} = \boldsymbol{i}_a$, then the power factor defined by (26) becomes $\lambda = 1$. It follows that the current component $\boldsymbol{i}_a$ in a three-wire three-phase system is the minimum rms value current that determines the active power P_p for a specified voltage.

The vectorial equations (30) and (31) are more effective than the separate application of the time-domain decomposition to the single phases, since they attain the optimal redistribution of the average phase powers for a specified total power.

Moreover, if the following equations were written for each phase, in spite of (30):

$$i_{am} = \frac{P_m}{V_m^2} \cdot v_m(t), \qquad m = a, b, c$$

v_m being the line voltages of the single phases, it would be obtained that

$$i_{aa} + i_{ab} + i_{ac} \neq 0$$

that has no meaning in three-wire systems. To have zero sum currents, the following equations [9], [12], [17] should be written:

$$i_{am} = \frac{P_a + P_b + P_c}{V_a^2 + V_b^2 + V_c^2} \cdot v_m(t), \qquad m = a, b, c$$

that, applying the Park transformation, leads to (30).

B. Frequency-Domain Decomposition

The frequency-domain decomposition of the current Park vector leads to a generalization to nonsymmetrical voltage conditions of the theories introduced by Czarnecki in [7]–[10].

The harmonic components $\boldsymbol{I}_k$ and $\boldsymbol{V}_k$[11] of vectors $\boldsymbol{i}(t)$ and $\boldsymbol{v}(t)$ respectively are obtained by (10). The current components can be divided into two groups: those with $k \in N_u$ and those with $k \in N_f$, where N_u is the set of harmonic components of the voltage vector $\boldsymbol{v}(t)$ and N_f is the set of harmonic components of current that does not include the harmonic components of the voltage vector $\boldsymbol{v}(t)$. This can be also expressed by the fact that $N_u \cap N_f = 0$.

For each term with $k \in N_u$, the complex ratio

$$\frac{\boldsymbol{I}_k}{\boldsymbol{V}_k} = G_k + jB_k \tag{34}$$

can be considered. Moreover, from (23) and (34), it follows:

$$P_p = \sum_{k \in N_u} G_k \cdot V_k^2 \tag{35}$$

$$Q_p = -\sum_{k \in N_u} B_k \cdot V_k^2. \tag{36}$$

It is then possible to define an equivalent conductance G_e so that

$$G_e = \frac{P_p}{V^2}.$$

The active current $\boldsymbol{i}_a$ defined by (30) can be rewritten as:

$$\boldsymbol{i}_a(t) = G_e \cdot \boldsymbol{v}(t) = G_e \cdot \sum_{k \in N_u} \boldsymbol{V}_k e^{jk\omega t}. \tag{37}$$

From (34) and (37), the following components of the current $\boldsymbol{i}(t)$ can be defined

$$\boldsymbol{i}_{ag}(t) = \sum_{k \in N_u} G_k \cdot \boldsymbol{V}_k e^{jk\omega t} \tag{38}$$

$$\boldsymbol{i}_s(t) = \boldsymbol{i}_{ag}(t) - \boldsymbol{i}_a(t) = \sum_{k \in N_u} (G_k - G_e) \cdot \boldsymbol{V}_k e^{jk\omega t} \tag{39}$$

$$\boldsymbol{i}_r(t) = j \sum_{k \in N_u} B_k \cdot \boldsymbol{V}_k e^{jk\omega t} \tag{40}$$

and, as far as the terms with $k \in N_f$ are concerned,

$$\boldsymbol{i}_f(t) = \sum_{k \in N_f} \boldsymbol{I}_k e^{jk\omega t}. \tag{41}$$

It can then be written:

$$\boldsymbol{i} = \boldsymbol{i}_a + \boldsymbol{i}_s + \boldsymbol{i}_r + \boldsymbol{i}_f. \tag{42}$$

In agreement with the definitions of [7], [9], the current $\boldsymbol{i}_s$ can be called the "scattering current" and the current $\boldsymbol{i}_r$ can be called the "reactive current."

The complex product between two complex functions of period T can be now defined as:

$$\{\boldsymbol{F}, \boldsymbol{H}\} = \frac{1}{T} \cdot \int_T \boldsymbol{F}(t) \cdot \boldsymbol{H}^*(t) \cdot dt = \boldsymbol{C}$$

$\boldsymbol{C}$ being a complex constant. If the two functions are expressed by their harmonic components $\boldsymbol{F}_k$ and $\boldsymbol{H}_k$, the complex product can be obtained by

$$\{\boldsymbol{F}, \boldsymbol{H}\} = \sum_k \boldsymbol{F}_k \cdot \boldsymbol{H}_k^*.$$

The above definitions, along with (35) and (36), allow the real and imaginary power to be written in terms of the defined current components:

$$\{\boldsymbol{v}, \boldsymbol{i}_{ag}\} = \sum_{k \in N_u} G_k \cdot V_k^2 = P_p \tag{43}$$

$$\{\boldsymbol{v}, \boldsymbol{i}_s\} = \sum_{k \in N_u} (G_k - G_e) \cdot V_k^2 = P_p - P_p = 0 \tag{44}$$

$$\{\boldsymbol{v}, \boldsymbol{i}_r\} = -j \sum_{k \in N_u} B_k \cdot V_k^2 = jQ_p \tag{45}$$

$$\{\boldsymbol{v}, \boldsymbol{i}_f\} = 0. \tag{46}$$

These current components are so related directly to the defined powers and not only to apparent powers as in [8] and [9].

Moreover, (45) gives a further physical meaning to the average value (24) of the imaginary Park power: If the load is linear, passive and time-invariant, Q_p is associated with the imaginary elements of (34) only.

The orthogonality between $\boldsymbol{i}_a$, $\boldsymbol{i}_s$, $\boldsymbol{i}_r$, and $\boldsymbol{i}_f$ components can be proven if the scalar product between each pair of them is null.

The scalar product between two complex functions of period T is defined as:

$$(\boldsymbol{F}, \boldsymbol{H}) = \frac{1}{T} \int_T \mathrm{Re}\,[\boldsymbol{F}(t) \cdot \boldsymbol{H}^*(t)] \cdot dt = \mathrm{Re}\,[\{\boldsymbol{F}, \boldsymbol{H}\}] = C$$

C being a scalar constant. If $\{\boldsymbol{F}, \boldsymbol{H}\} = 0$, then $(\boldsymbol{F}, \boldsymbol{H}) = 0$.

It follows from (37) and (44):

$$(\boldsymbol{i}_a, \boldsymbol{i}_s) = G_e(\boldsymbol{v}, \boldsymbol{i}_s) = 0$$

and (37) and (45):

$$(\boldsymbol{i}_a, \boldsymbol{i}_r) = G_e(\boldsymbol{v}, \boldsymbol{i}_r) = 0$$

and:

$$(\boldsymbol{i}_a, \boldsymbol{i}_f) = (\boldsymbol{i}_s, \boldsymbol{i}_f) = (\boldsymbol{i}_r, \boldsymbol{i}_f) = 0$$

since none of the harmonic components of $\boldsymbol{i}_f$ are present in the other currents by definition.

Because of the orthogonality, the following equation can be written for the rms values:

$$I^2 = I_a^2 + I_s^2 + I_r^2 + I_f^2. \tag{47}$$

In the particular case of symmetrical voltage (although nonsinusoidal) the above-defined current components $\boldsymbol{i}_a$, $\boldsymbol{i}_s$, and $\boldsymbol{i}_r$ represent the Park vectors of the currents i_a, i_s, and i_r defined by Czarnecki [9], [10].

Since the Park theory deals with positive and negative sequence components as if they were different harmonic

[11] According to the fact that (10) represents a complex Fourier series, index k ranges from $-\infty$ to $+\infty$, as already stated.

components, the current $\boldsymbol{i}_f$ represents the extension to three-phase systems of the current i_g defined in [7][12].

The relationship between the imaginary Park power and the decomposition (29) of the apparent power can be obtained again from (47) if a further decomposition of $\boldsymbol{i}_r$ is considered.

If the equivalent susceptance is defined:

$$B_e = \frac{Q_p}{V^2}$$

the current

$$\boldsymbol{i}_q(t) = -jB_e \cdot \boldsymbol{v}(t) \tag{48}$$

can be defined, as well as the current

$$\boldsymbol{i}_{rs}(t) = \boldsymbol{i}_r(t) - \boldsymbol{i}_q(t). \tag{49}$$

If the rms value I_q of $\boldsymbol{i}_q(t)$ is considered, it follows:

$$Q_p = V \cdot I_q. \tag{50}$$

It can be easily proven (in just the same way as that followed in the previous section for the current $\boldsymbol{i}_a$) that the current $\boldsymbol{i}_q$ is the minimum rms value current required to give the average value Q_p of the imaginary Park power. The current $\boldsymbol{i}_{rs}$ assumes then the meaning of "reactive scattering" current.

Moreover, the orthogonality between $\boldsymbol{i}_q$ and $\boldsymbol{i}_{rs}$ can be proven. In fact it can be written, from (45) and (48):

$$\begin{aligned}\{\boldsymbol{v}, \boldsymbol{i}_{rs}\} &= \{\boldsymbol{v}, \boldsymbol{i}_r\} - \{\boldsymbol{v}, \boldsymbol{i}_q\} \\ &= jQ_p - jQ_p = 0\end{aligned} \tag{51}$$

and, from (48) and (51):

$$\{\boldsymbol{i}_q, \boldsymbol{i}_{rs}\} = -jB_e \cdot \{\boldsymbol{v}, \boldsymbol{i}_{rs}\} = 0. \tag{52}$$

It follows that

$$I_r^2 = I_q^2 + I_{rs}^2 \tag{53}$$

and, from (47), that

$$I^2 = I_a^2 + I_s^2 + I_q^2 + I_{rs}^2 + I_f^2. \tag{54}$$

From (33), (50), and (54), the apparent power decomposition (29) is obtained, where

$$D_p^2 = V^2 I_s^2 + V^2 I_{rs}^2 + V^2 I_f^2. \tag{55}$$

Equation (29) gives indication on how to decrease the apparent power S and to increase the power factor λ. In fact, the average value Q_p of the Park imaginary power can be completely compensated by means of static compensators without employing energy storage elements (as already stated) and without affecting the other power components.[13]

The knowledge of Q_p is the only requirement to achieve this compensation and it can be measured easily, with no need for the frequency-domain analysis.

VI. Conclusions

A new method has been proposed for the definition of active and nonactive power components in three-phase systems under nonsinusoidal conditions.

The method proves itself more attractive than other proposed ones since it is not a mere extension of methods employed in single-phase systems, but comes from the application of a quite powerful and synthetic mathematical tool specially studied for the representation of three-wire three-phase systems in any possible condition: the Park transformation and the Park vectors.

It was proven that the application of this method leads to the definition of two quantities, the real and the imaginary power, that are measurable in a quite simpler way than those proposed by other theories, that satisfy to all properties typical of the electrical power and that are directly related, under sinusoidal and balanced conditions, to the active and reactive powers.

The generality of this method was then proved, showing how some of the most significant theories proposed by other authors, based on time and frequency-domain decomposition of currents, can be directly extended to three-phase systems employing this method.

[12]It is possible to attain a further decomposition of $\boldsymbol{i}_f$ in order to reveal explicitly the effect of the circuit asymmetry on the source current (as in [9], [10]). However, the decomposition introduced in [9] does not seem to be very useful, since it is valid only when the supply voltage is symmetrical. After all, it has been proven [14] that sequence and harmonic decompositions have similar formal properties so that it is quite meaningful to consider them together.

[13]This can be easily obtained if the current injected by the static compensator is the $-\boldsymbol{i}_q(t)$.

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