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The Input Impedance of a Slit Antenna

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#1 Duality property between electric current/magnetic current and electric field/magnetic field

Let us consider a space of constant permittivity ε and constant permittivity μ having electric current sources and magnetic current sources with surfaces of $S_1, S_2 \cdots$ and $S_1', S_2' \cdots$, respectively. In the space except for these surfaces, following Maxwell's equations hold.

$$\left.\begin{array}{ll} \nabla\times\boldsymbol{E} + j\omega\mu\boldsymbol{H} = -\boldsymbol{J}', & \nabla\cdot\boldsymbol{H} = \dfrac{\rho'}{\mu} \\ \nabla\times\boldsymbol{H} - j\omega\varepsilon\boldsymbol{E} = \boldsymbol{J}, & \nabla\cdot\boldsymbol{E} = \dfrac{\rho}{\varepsilon} \end{array}\right\}, \tag{1}$$

where $\boldsymbol{J}$, $\boldsymbol{J}'$, ρ and ρ' are the electric current, the magnetic current, the electric charge and the magnetic charge, respectively. They satisfy the equations of continuity given by

$$\nabla\cdot\boldsymbol{J} = -j\omega\rho, \quad \nabla\cdot\boldsymbol{J}' = -j\omega\rho'. \tag{2}$$

Electromagnetic fields satisfy the boundary conditions given by

$$\left.\begin{array}{ll} \boldsymbol{E}\times\boldsymbol{n} = 0, \quad \boldsymbol{H}\cdot\boldsymbol{n} = 0 & \text{on } S_1, S_2\cdots \\ \boldsymbol{H}\times\boldsymbol{n} = 0, \quad \boldsymbol{E}\cdot\boldsymbol{n} = 0 & \text{on } S_1', S_2'\cdots \end{array}\right\}, \tag{3}$$

where $\boldsymbol{n}$ is the unit normal vector on these surfaces.

When the currents and the charges are given by

$$\begin{cases} \boldsymbol{J} = \boldsymbol{N}, \\ \rho = n, \end{cases} \begin{cases} \boldsymbol{J}' = \boldsymbol{M} \\ \rho' = m \end{cases} \text{where} \begin{cases} \nabla\cdot\boldsymbol{N} = -j\omega n \\ \nabla\cdot\boldsymbol{M} = -j\omega m \end{cases}, \tag{4}$$

and the electromagnetic fields are given by

$$\boldsymbol{E} = \boldsymbol{F}, \quad \boldsymbol{H} = \boldsymbol{G}, \tag{5}$$

the following equations hold

$$\left.\begin{array}{l} \nabla\times\boldsymbol{F} + j\omega\mu\boldsymbol{G} = -\boldsymbol{M} \\ \nabla\times\boldsymbol{G} - j\omega\varepsilon\boldsymbol{F} = \boldsymbol{N} \end{array}\right\}, \tag{6}$$

and

$$\left.\begin{array}{ll} \boldsymbol{F}\times\boldsymbol{n} = \boldsymbol{G}\cdot\boldsymbol{n} = 0 & \text{on } S_1, S_2\cdots \\ \boldsymbol{G}\times\boldsymbol{n} = \boldsymbol{F}\cdot\boldsymbol{n} = 0 & \text{on } S_1', S_2'\cdots \end{array}\right\}. \tag{7}$$

Eq. (6) can be expressed by changing the order of equations as

$$\left.\begin{array}{l}\nabla\times(-\gamma\boldsymbol{G})+j\omega\mu(\gamma^{-1}\boldsymbol{F})=-(\gamma\boldsymbol{N})\\ \nabla\times(\gamma^{-1}\boldsymbol{F})-j\omega\varepsilon(\gamma\boldsymbol{G})=(-\gamma^{-1}\boldsymbol{M})\end{array}\right\}. \tag{8}$$

By making similar modification and comparing with eqs. (1)-(3), the electric and the magnetic sources are expressed as

$$\left.\begin{array}{l}\boldsymbol{J}=-\gamma^{-1}\boldsymbol{M},\quad \boldsymbol{J}'=\gamma\boldsymbol{N}\\ \rho=-\gamma^{-1}m,\quad \rho'=\gamma n\end{array}\right\}. \tag{9}$$

The electromagnetic fields for the case that the perfect electric conductor and the perfect magnetic conductor with the perfect magnetic conductor and the perfect electric conductor, respectively, are given by

$$\boldsymbol{E}=-\gamma\boldsymbol{G},\quad \boldsymbol{H}=\gamma^{-1}\boldsymbol{F}. \tag{10}$$

#2 Duality property between plates and holes

When there are a perfectly electric conducting plane S on a part of a plane $x=0$, other part of the plane $\overline{S}$ on the plane $x=0$, and electric current expressed by

$$\left.\begin{array}{ll}\boldsymbol{J}(x)=\boldsymbol{N}(x), & x>0\\ \boldsymbol{J}(x)=\boldsymbol{N}(-x), & x<0\end{array}\right\}. \tag{11}$$

If the electromagnetic fields are given by

$$\left.\begin{array}{l}\boldsymbol{E}(x)=\boldsymbol{F}(x)\\ \boldsymbol{H}(x)=\boldsymbol{G}(x)\end{array}\right\},\quad x>0,\quad \left.\begin{array}{l}\boldsymbol{E}(x)=\boldsymbol{F}'(x)\\ \boldsymbol{H}(x)=\boldsymbol{G}'(x)\end{array}\right\},\quad x<0 \tag{12}$$

these fields are electrically symmetric (E-symmetry) as

$$\left.\begin{array}{r}-F_x(-x)=F_x'(x)\\ F_y(-x)=F_y'(x)\\ F_z(-x)=F_z'(x)\end{array}\right\},\quad \left.\begin{array}{r}G_x(-x)=G_x'(x)\\ -G_y(-x)=G_y'(x)\\ -G_z(-x)=G_z'(x)\end{array}\right\}. \tag{13}$$

In this case, the surface $\overline{S}$ is equivalent to a perfect magnetic conductor and the fields do not change when we insert a perfect magnetic conductor on $\overline{S}$. Therefore, the fields in the spaces divided by a plane of $x=0$ are independent each other. Next, we exchange the perfect electric conductor of S for the perfect magnetic conductor and the perfect magnetic conductor of $\overline{S}$ for the perfect electric conductor. The electric current is replaced with magnetic current as

$$\boldsymbol{J}'(x)=\gamma\boldsymbol{N}(x)\ \ (x>0),\quad \boldsymbol{J}'(x)=-\gamma\boldsymbol{N}(-x)\ \ (x<0). \tag{14}$$

Using the results in #1, the electromagnetic fields can be expressed by

$$\left.\begin{aligned} \boldsymbol{E}(x) &= -\gamma \boldsymbol{G}(x) \\ \boldsymbol{H}(x) &= \gamma^{-1} \boldsymbol{F}(x) \end{aligned}\right\} (x > 0), \quad \left.\begin{aligned} \boldsymbol{E}(x) &= \gamma \boldsymbol{G}'(x) \\ \boldsymbol{H}(x) &= -\gamma^{-1} \boldsymbol{F}'(x) \end{aligned}\right\} (x < 0). \quad (15)$$

Since these fields are also symmetric (E-symmetry), the electromagnetic fields do not change when the perfect magnet conductor on S is removed. Thus, the problem of the symmetric electric current distribution with perfect electric conductor reduces the problem of antisymmetric perfect magnetic current distribution with the hole.

#3 Input impedance of a slit antenna

A slit antenna fed by an electric source shown in Fig. 1 can be considered to be the slit antenna fed by a magnetic source shown in Fig. 2. Let us consider a planar dipole antenna shown in Fig. 3, which has the duality property with the slit antenna of Fig. 2. Let the input impedance of the planar dipole be *Z'*, and the current and the voltage be *I'* and *V'*. Since the electric current is symmetric, we obtain the following equations using eq. (12).

$$V' = \int_d^c \boldsymbol{E} \cdot d\boldsymbol{S} = \int_d^c \boldsymbol{F} \cdot d\boldsymbol{S}$$

$$I' = \oint_{aba} \boldsymbol{H} \cdot d\boldsymbol{S} = 2\int_a^b \boldsymbol{H} \cdot d\boldsymbol{S} = 2\int_a^b \boldsymbol{G} \cdot d\boldsymbol{S}$$

$$\therefore Z' = \frac{V'}{I'} = \frac{1}{2} \frac{\int_d^c \boldsymbol{F} \cdot d\boldsymbol{S}}{\int_a^b \boldsymbol{G} \cdot d\boldsymbol{S}}$$

Since the magnetic current shown in Fig. 2 is antisymmetric and the geometry of Fig. 3 has the duality property with that of Fig. 2, the current *I*, the voltage *V* and the impedance *Z* between *a* and *b* are expressed by using eq. (15) as

$$V = \int_a^b \boldsymbol{E} \cdot d\boldsymbol{S} = -\gamma \int_a^b \boldsymbol{G} \cdot d\boldsymbol{S}$$

$$I = \oint_{dcd} \boldsymbol{H} \cdot d\boldsymbol{S} = \frac{1}{\gamma} \int_c^d \boldsymbol{F} \cdot d\boldsymbol{S} + \frac{1}{\gamma} \int_d^c (-\boldsymbol{F}) \cdot d\boldsymbol{S} = 2\frac{1}{\gamma} \int_c^d \boldsymbol{F} \cdot d\boldsymbol{S},$$

$$\therefore Z = \frac{V}{I} = \frac{\gamma^2}{2} \frac{\int_a^b \boldsymbol{G} \cdot d\boldsymbol{S}}{\int_d^c \boldsymbol{F} \cdot d\boldsymbol{S}}. \quad (17)$$

From eqs. (16) and (17), we have

$$Z = \frac{\gamma^2}{4} \frac{1}{Z'} = \frac{(60\pi)^2}{Z'}. \quad (18)$$

Eq. (18) has been derived using the electromotive force method based on the idea of radiating power, but we can derive eq. (18) directly using the relation between the current and the voltage. It has been found that the input impedance of an antenna, which is composed of two conducting plates on quarters of infinite plane as shown in Fig. 4, is $60\pi\,\Omega$. It is concluded that the relation between the impedance matrix [Z] of the slit antenna and that of [Z'] of the planar antenna having the duality property with the slit antenna can be expressed by following equation.

$$[Z] = (60\pi)^2 [Z']^{-1} \tag{19}$$

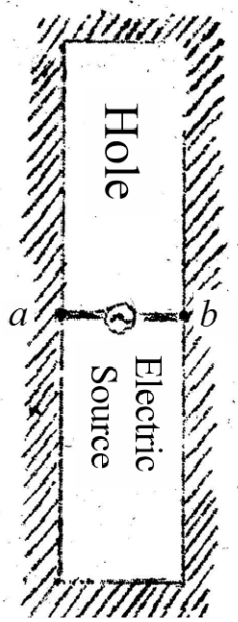

Fig. 1

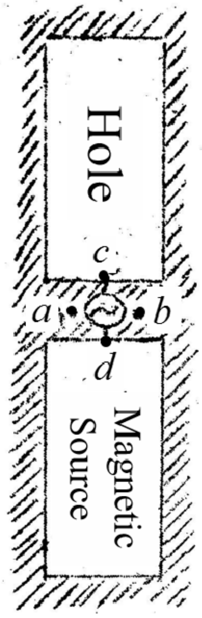

Fig. 2

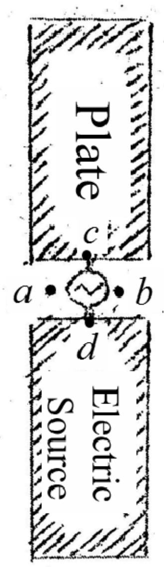

Fig. 3

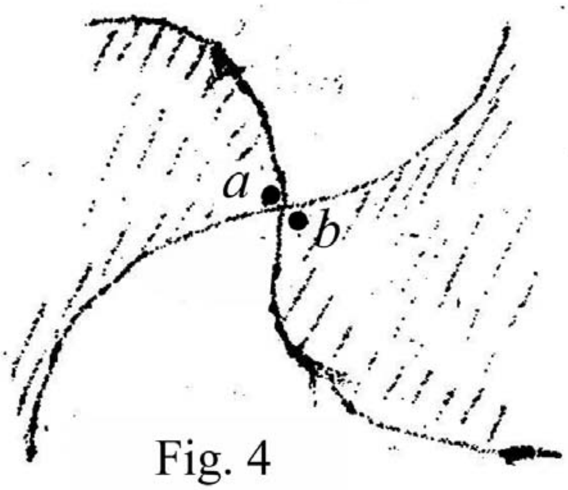

Fig. 4