ON THE SYSTEMS OF ELECTRIC UNITS.

BY PROF. M. ASCOLI, *President and Delegate of the Associazione Elettrotecnica Italiana.*

1). I think that the International Electrical Congress should take cognizance of what has been done on the subject of electrical units since the last Congress. Therefore, I believe that a short report on the work done, especially in Italy, on this important question may not be without interest.

What I intend to say has nothing to do with any change which may be proposed of the value of the practical standard used at present. This paper is intended to deal with the fundamental theory of units.

2). In the equations of electromagnetism on which the definition of units is founded, we have several coefficients, to some of which we give particular values so as to establish particular systems of units. But I think it will be more useful in order to prevent any misunderstanding to keep at first all the coefficients in the formulæ, leaving to these their generality.

It is preferable, I think, in deducing the units to start from the old expressions of the laws of different kinds of mutual action between electric and magnetic quantities; that is to say, electrostatic, magnetic, electromagnetic, and electrodynamic actions.

We have thus four equations, one side of which is a force, expressing the laws of Coulomb, Laplace, and Ampere:

$$(1) \qquad f=\frac{1}{\alpha}\frac{e^2}{r^2} \qquad f=\frac{1}{\beta}\frac{m^2}{r^2} \qquad f=\frac{1}{\gamma}\frac{mids}{r^2}\sin\omega$$

$$f=\frac{1}{\delta}\frac{ii'\,ds\,ds'}{r^2}\left(\cos\epsilon-\frac{3}{2}\cos\theta\cos\theta'\right)$$

where the symbols have a well-known signification, and α, β, γ, δ, are special coefficients.

A fifth equation is

$$i=\frac{e}{t}$$

which can be said to express the equivalence between the current defined by electromagnetic action and the current defined by convection.

Many other equations frequently used may be considered as definitions of so many other magnitudes. For example

$$
(2)\qquad
\begin{aligned}
\text{Energy} &= eV && \text{defines the potential } V\\
V &= iR && \text{defines the resistance } R\\
e &= CV && \text{defines the capacity } C\\
\text{Energy} &= \tfrac{1}{2}Li^2 && \text{defines the self-induction } L.
\end{aligned}
$$

In the preceding five equations we have seven magnitudes not having a geometrical or mechanical character, or, in other words, not depending only on the three fundamental magnitudes: length, mass, and time. These seven magnitudes are e, m, i, α, β, γ, δ. We have five equations between seven quantities; two of them must, therefore, be chosen in order to have the others determined; and the choice must be an arbitrary one, until some new physical laws are discovered.

It has been, therefore, a misconception to suppose that length, mass, and time were sufficient to define the electric units. On the contrary, the fundamental magnitudes must be five in number. Three of them can be l, m, t; the other two can be chosen in any way among the seven above-mentioned quantities, or others connected with them, by some known equations. It is not at all necessary to choose these two quantities among the coefficients α, β, γ, δ.

It has been suggested that it would be possible for new laws to be discovered in the future, so that even the coefficients above mentioned might be expressed in terms of l, m, and t. But in the present state of the science, this suggestion is entirely arbitrary. Mr. Fessenden, for instance, following Maxwell's conceptions, tried to assume that of the two coefficients α and β, the first could be a density, the second the reciprocal of a pressure; but Fessenden's arguments are at the utmost valuable only in suggesting the hypothesis of the proportionality between the said quantities, but not the hypothesis of their equality. In this way a new constant of unknown nature is introduced.

3). It is easy to deduce from the five equations above quoted[1] that the four coefficients, α, β, γ, δ, are connected by the two relations

1. See Prof. Somigliana, "Sulle unita elettriche e magnetiche." *Rendimenti* dell' Institute Lombardo. January, 1900.

(3) $$\frac{\gamma^2}{\alpha\beta} = v^2 \quad \frac{\delta}{\alpha} = v^2$$

in which v represents a velocity that experiment proves to be dependent upon the nature of the matter occupying the region considered, and equal to the velocity of light in the same. Only two of the four coefficients can be chosen at will; that is to say two quantities are sufficient to define the electromagnetic properties of the surrounding matter, of the ether, for instance.

In the electrostatic system if we assume $\alpha = 1$, $\gamma = 1$, it follows that $\frac{1}{\beta} = v^2$, $\delta = v^2$; in the electromagnetic system, if $\beta = 1$, $\gamma = 1$, it follows that $\frac{1}{\alpha} = v^2$, $\delta = 1$. If we suppose $\alpha = 1$, $\beta = 1$, as in a Hertzian system, we have $\gamma = v$, $\delta = v^2$.

Any system, provided that it satisfies the above conditions, is a rational one. It would be, therefore, preferable to choose a different word to indicate the system which gets rid of 4π from the formula of electromagnetism, suggested by Mr. Heaviside.

To obtain the rationalization, in the Heaviside sense, we must put $\gamma = 4\pi$ in the expression of the m.m.f. $(g = \frac{4\pi}{\gamma} i)$ which follows from the third of the above equations. We have in this case, from

(3) $$\frac{(4\pi)^2}{\alpha\beta} = v^2.$$

If together with this rationalization we wish also to keep unaltered the unit of electric quantity as chosen in the electromagnetic system, as was proposed by Fessenden, we must put $\frac{1}{\alpha} = v^2$, as it is in this system; it follows that $\beta = (4\pi)^2$, as the value of the permeability of the standard medium, as Mr. Fessenden stated, notwithstanding the remedy of Professor Fleming. For this example we have a proof of the utility of the above method.

4). But the so-called rationalization, notwithstanding the proposals of Fleming and Fessenden, introduces some units that are not used in practice. So we will find it necessary to retain five different systems, unless we accept the proposal of Mr. Fessenden and assume the c.g.s. units also for practical purposes; but this change would itself be undoubtedly so objectionable, that I can hardly see how the proposal could be accepted.

Professor Giorgi of Rome took up the question more than three years ago in order to construct a system, rational in the Heaviside meaning, but at the same time an absolute one; that is to say, depending upon a minimum number of fundamental units and having the advantage, essential for practical use, of keeping the units of the actual practical system for the measurement of the most important magnitudes.

The proposals of Mr. Giorgi were presented and discussed before the Associazione Elettrotecnica Italiana at the general meeting of October, 1901 (Rome). After this discussion a committee was appointed (Professors Ascoli, Donati, Grassi, Lombardi, Róiti), who presented a report also on behalf of the Italian Physical Society, at the general meeting of the Associazione in October, 1902 (Turin). The committee in this report expressed the opinion that the Giorgi system had the necessary characters which would entitle it to be substituted for the present systems, and recommended that the new system be brought before an international electrical congress.

In the meantime Mr. Giorgi has published some explanatory notes; the details of the system were given in the technical periodicals of Europe and America. It was also presented and discussed before the London Physical Society and a report of this discussion can be found in the London *Electrician*, in 1902. Professor Robertson subsequently returned in the same journal to the Giorgi proposals, and several authors, who took part in the discussion which ensued, seemed to have entirely forgotten the remarkable preceding work. Mr. Emde in Germany in an interesting paper on electric units read before the Elektrotechnische-Verein in Berlin some months ago (see E. T. Z.) discussed at length and favorably the Giorgi system.

The Giorgi system is a rational one (in the Heaviside sense); that is to say, it assumes the relation $\gamma = 4\pi$, and the coefficient γ is, therefore, one of the two magnitudes chosen in an arbitrary way. As regards the second outstanding magnitude, Mr. Giorgi proposed not one of the other coefficients, but one of the practical electrical units used at present; for instance the Chicago international ampere or ohm. In this case the ampere (or ohm) will no longer remain a unit theoretically defined, but it would become a fundamental arbitrary unit. In the same way, the meter is no more considered the ten-millionth of the terrestrial quadrant, but it is the length of the platinum bar existing in Paris which very approximately corresponds to the primitive definition.

The measurements of the earth, which at first were intended for the definition of the meter, now express in meters the length of the quadrant. What would be now the result of measurements made for the determination of the absolute electric units, which required such long and hard labor? As has already been stated, among the quantities which are no longer arbitrary, we have the two coefficients, α and β; that is, the electric and magnetic constants of the standard medium, of the ether, for instance. It would perhaps be better to change, in the work of the Chicago congress, the value of these constants instead of changing the value of the ohm previously employed.

5). It is easy now to see that, independently of the fundamental units of length, mass, and time, the units of e.m.f., resistance, capacity, inductance — that is, the most important ones in practical use — are not at all changed if we keep the same unit of time, energy (or power) and of electrical current (or resistance). In fact, this is plainly shown by the equations (3). For this reason we are free to choose any units of length and mass, provided that the unit of energy resulting from them remains the present joule; that is 10,000,000 ergs of the c.g.s. system. We can, therefore, choose as unit of length 10^n cm, and as unit of mass, 10^m grams, provided that $2n+m=7$. We can take, for example, $n=2$, $m=3$, that is, take the meter and kilogram as units of length and mass. These units seem to be very appropriate because the Paris standards are precisely the meter and kilogram.

We have, in this way, the advantage of establishing a new system of rational units, keeping the practical system now used, and of reducing this system to an absolute one with very convenient fundamental units (meter, kilogram, and second).

Some may regret to give up the c.g.s. system, but it can be observed that the c.g.s. system is already partially abandoned in practice and that it will always retain its historical value because the arbitrary unit of current (or resistance) that we would choose will have its origin in the old system.

If we suppose that the international ampere (or ohm) corresponds exactly to the theoretical definition depending upon the c.g.s. system, the electric and magnetic constant of the ether would be

$$\alpha = \frac{1}{36\pi} 10^{-9} \quad \beta = 4\pi \, 10^{-7}.$$

But in fact these constants are affected by the errors of observation made in the absolute measurements; if the 4π enters in the expression of α and β, it occurs only from a historical reason, because the arbitrary unit is chosen very near to the value of a non-rational unit.

In conclusion, I believe that the Giorgi system must be preferred to the other rational systems which have been hitherto proposed. No difficulty of a legal kind exists against it, because none of the units accepted by the government are charged; on the contrary it prevents any change in the future. I do not think that it is necessary at present to introduce officially the new system, the more so as any one can use it without trouble of any sort; but I believe that the Congress should take cognizance of it, and put it on an official plane with other systems which have been proposed and may be proposed in the future. Especially I would like to call on it the attention of professors teaching electrical science.

APPENDIX.

PROPOSALS CONCERNING ELECTRICAL AND PHYSICAL UNITS.

BY PROF. G. GIORGI.

It is suggested that the existing system of practical units may be completed as follows, thus making an absolute system of practical units:

1. *Concrete Electrical and Magnetic Units.*

Besides the existing units, ohm, coulomb, volt, farad, henry, and ampere, the following ones are proposed:

For *m.m.f.*, the ampere (already practically used under the name of ampere-turn).

For *magnetic flux*, the product of one volt by one second, which may be called the *weber* (as proposed by the British association).

For *magnetic inductance* (permeance, that is $\frac{\text{flux}}{\text{m.m.f.}}$), the henry (already existing as the unit of self-induction).

These, together with their reciprocals, make a complete and self-consistent system of electrical and magnetic concrete units. They may be combined with the following:

2. *Mechanical Units.*

For *length*, the meter.
For *mass*, the kilogram.
For *time*, the second.

Thence —

For *power*, the watt.
For *work*, the joule, etc., etc.

3. *Electrical and Magnetic Specific Units.*

No name for any specific unit is proposed. Instead of having specific units ready made, it is preferable to make them by referring the concrete units to any unit of length, area, volume, which may be preferable, according to the circumstances of a case; thus, *volt/m*, or *volt/mm*, or *volt/inch* as it may be desirable.

When the meter, kilogram, and second are taken as fundamental, specific units of the absolute practical system result as follow:

Amp./m for magnetic force (magnetic field intensity, gradient of magnetic potential).

Volt/m for electric force (electric field intensity, gradient of electric potential).

Weber/m² for magnetic induction (magnetic displacement, magnetic flux per unit area).

Coulomb/m² for electric induction (electric displacement, electric flux per unit area).

Henry/m for magnetic inductivity (permeability, magnetic constant of a medium).

Remark.— The magnetic constant of free ether becomes $\mu_0 =$ 0.000,001,256 henry/m.

Farad/m for electric inductivity (dielectric power, electrostatic constant, ratio of electric displacement to electric force).

Remark.— The electric constant of free ether becomes $k_0 =$ 0.000,000,000,008,842 farad/m.

Results.

In this manner we obtain an *absolute system of practical units*, which is independent of both the c.g.s. electrostatic system and the c.g.s. electromagnetic system, and does not interfere with either.

As fundamental units of this system there may be taken, the *meter*, the *kilogram*, the *second*, and the *ohm* (the latter to be defined by the practical standard adopted by the Congress of 1903, or by the standard kept at the Board of Trade in London).

This system is "rationalized" (in Mr. O. Heaviside's signification); that is, is free from any unnecessary 4π. In this system, electric current is identified with m.m.f.

This system is neither electrostatic nor electromagnetic, because neither the electric nor the magnetic constant of free ether is assumed as a fundamental unit.

This system is completely dualistic, all units having a magnetic and an electric signification at the same time, which halves the number of units needed; all electric and magnetic formulæ are identical.

All units, fundamental and derived, are of convenient size.

The system may be called the *absolute practical system.* Its units may be called *absolute practical units.*

Concerning Practical Use.

The system consists entirely of units already in practical use.

Practicians are not required to make any change, nor to learn anything new. They are simply to be instructed that their present units may also be used as absolute ones, thereby making the c.g.s. systems unnecessary in their calculations.

The necessity of making conversion of units is thus avoided (see Note C).

Concerning Scientific Use.

Neither the c.g.s. electrostatic nor the c.g.s. electromagnetic system is touched. Scientists will be free to use any one of these systems, without modification, or to substitute for them the absolute practical system, with the advantage of simplified and rationalized formulæ; agreement with practical use; units of convenient size; dimensions simple, without fractional exponents; fundamental units independent of absolute measurement; no distinction to be made between electrostatic and electromagnetic calculations.

Theoretical Grounds.

The theoretical grounds on which the absolute practical system is founded are fully set forth and discussed in the papers mentioned in Note A.

The point of fundamental importance to be kept in view is the following:

In order to derive electric and magnetic units from mechanical units, a fourth fundamental or independent unit is necessary. In the c.g.s. electrostatic and in the c.g.s. electromagnetic systems, the fourth unit assumed is respectively the electrostatic or the magnetic constant of free ether; but this has many disadvantages. In the absolute practical system, the fourth unit is the *ohm.*

Of course, when any electric or magnetic unit is arbitrarily chosen, all others are deduced from it.

Note A.

History.

1). G. Giorgi.—"Unita Razionali di Elettromagnetismo," read before the general meeting of the Italian Association of Electrical Engineers, October, 1901, in Rome. See *Atti* dell' Associaz. Elettr. Italiana, 1901, p. 402; *L'Elettricista,* 1901, December; *L'Elettricita,* 1901; *L'Industria,* 1901 (+); *Il Nuovo Cimento,* 1902. See also abstracts in *Science Abstracts,* in *l'Eclairage Electrique,* etc.

2). Discussion of Same.—See report of said meeting, in *Atti* dell' A. E. T., 1901.

3). G. Giorgi.—"Rational units of electromagnetism," read before the Physical Society of London, on May 27, 1902.

4). Discussion of Same.

5). Prof. Donati.—Report on G. Giorgi's proposals. See *Nuovo Cimento,* 1902.

6). Prof. Ascoli.—Sul Sistema di Unita Proposto dall' Ing. Giorgi, read before the Congress of the Societa Italiana di Fisica, held at Brescia in September, 1902.

7). Discussion of Same. See *Nuovo Cimento,* 1902.

8.) G. Giorgi.—"Il Sistema Assoluto M. Kg. S. Read before the A. E. T., May 2, 1902. See *Atti* dell' A. E. T., October, 1902; *L'Elettricista,* 1902, etc.

9). Report of Committee, appointed by the Associazione Elettrotecnica Italiana, and by the Societá Italiana di Fisica, consisting of Prof. Grassi, Prof. Ascoli, Prof. Roiti, Prof. Lombardi, Prof. Donati; read by Prof. M. Ascoli at the general meeting of the Italian Electrical Association, held in Turin, November, 1902; also discussion of the same. See report of the meeting in *Atti* dell' A. E. T., 1902.

10). G. Giorgi.—"I Fondamenti della Teoria delle Grandezze Elettriche," read before the said Congress. See *Atti* dell' A. E. T., 1903. See also abstracts in *Science Abstracts* and elsewhere.

11). G. Giorgi.—"Le Formole Teoriche di Electtricita," read before the A. E. T., Dec. 15, 1902. See *Atti* dell' A. E. T., 1903.

12). G. Giorgi.—Notazioni e simboli Elettrici. See *Atti* dell' A. E. T., 1903.

Note B.

List of Units of the Absolute Practical System.

1.) *Mechanical.*

Magnitudes.	Absolute practical units.
Length	m
Area	m^2
Volume	m^3
Time	sec
Frequency	sec^{-1}
Velocity	m/sec
Acceleration	m/sec^2
Mass	kg
Density	kg/m^3
Force	(...... no name exists)
Torque	joule
Energy	joule
Power	watt

2). *Electrical.*

Magnitudes.	Absolute practical units.
Quantity of electricity	coulomb
Electric displacement	coulomb/m^2
Electric current	amp.
E.m.f	volt
El. force	volt/m
El. conductance	mho
El. conductivity	mho/m
El. resistance	ohm
Capacity	farad
El. inductivity (= specific capacity, or electric constant of a medium)..........................	farad/m
Coefficient of self-induction	henry

3). *Magnetic.*

Magnitudes.	Absolute practical units.
Quantity of magnetism (flux)	weber
Magnetic induction	weber/m^2
Magnetic current $\left(=\frac{d\psi}{dt}\right)$	volt
M.m.f.	ampere
Magnetic force	amp./m
Magnetic inductance $\left(= \text{permeance} = \frac{flux}{M.M.F.}\right)$,	henry
Magnetic inductivity (= magnetic constant of a medium, permeability)	henry/m
Magnetic reluctance	henry^{-1}

NOTE C.—METHOD OF APPLICATION.

To Calculate the Capacity of the Earth in Farads.

a). Following the methods hitherto used.	b). Using the absolute practical system.
Radius of the earth $r = 6\times10^8$cm	Radius of the earth $r = 6\times10^6 m$
Dielectric constant of free ether $k = 1$ (*electrostatic system*)	Dielectric constant of free ether $k = 88\times10^{-13}$ farad/m
Capacity of the earth, in c.g.s. electrostatic units $(K) = \frac{r}{k} = 6\times10^8$	Capacity of the earth $K = 4\pi kr = 67\times10^{-5}$ farad
Coefficient for converting electrostatic into electromagnetic value $v^2 = 9\times10^{20}$	
Capacity of the earth in c.g.s. electromagnetic units $[K] = \frac{(K)}{v^2} = \frac{6}{9}10^{-12}$	
Coefficient for converting c.g.s. value into practical value $e = 10^9$	
Capacity of the earth in farads $K = e[K] = 67\times10^{-5}$ farad	