Adapted Optimization Criterion for FDM-based DMT-ADSL Equalization

Jean-François Van Kerckhove and Paul Spruyt
Alcatel Research Division
Francis Wellesplein 1, B-2018 Antwerp, Belgium
Fax: + 32 3 240 9932 Tel: + 32 3 240 7743
E-mail: spruytp@btmaa.bel.alcatel.be

Abstract : The ANSI T1E1.4 Committee finalized the standardization of the Asymmetric Digital Subscriber Loop system and the usage of Discrete Multi-Tone modulation as the ADSL line coding scheme. In order to separate the upstream channel from the downstream channel, the Committee accepted two approaches as vendor options: Echo-Cancellation and Frequency Division Multiplexing. In this paper we want to illustrate the impact of the FDM approach on the equalization problem. The unsatisfactory performance of formerly known algorithms is described. A new, simple and near-optimal equalization optimization criterion that is specifically adapted to FDM-based DMT-ADSL is proposed.

1. INTRODUCTION

In June 1995, the ANSI T1E1.4 Committee finalized the standardization of the Asymmetric Digital Subscriber Loop (ADSL) system [1]. ADSL provides, on top of the analog POTS, down- and upstream bitrates up to approximately 7 Mbit/s and 640 kbit/s respectively over a single unconditioned twisted-wire pair of the telephone network. This asymmetric character of ADSL is especially well suited for the transport of multimedia services like Video on Demand, Internet Access and Interactive Games where the upstream channel is mainly used for remote control or interaction purposes.

For the transport of multi Mbit/s communication over unconditioned twisted-wire pair lines, the ANSI T1E1.4 Standardization Committee selected the Discrete Multi-Tone (DMT) modulation technique as the line coding scheme for ADSL [1]. The basic principle of DMT is to transmit the information bits in parallel over multiple carriers [2] (figure 1).

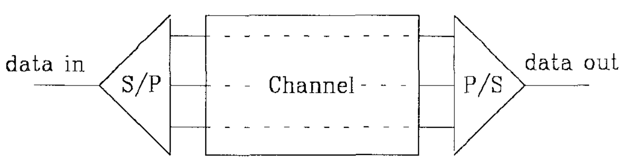

figure 1

For the separation of the up- and downstream transmission, two bandwidth allocation policies were accepted. The first one uses overlapping spectra for up- and downstream transmission and applies Echo-Cancelling : EC-based ADSL. The second option uses Frequency Division Multiplexing in which case no tones are shared by the up- and downstream transmission : FDM-based ADSL. The latter is depicted in figure 2. Notice that the lowest carriers are not modulated to avoid interference with POTS.

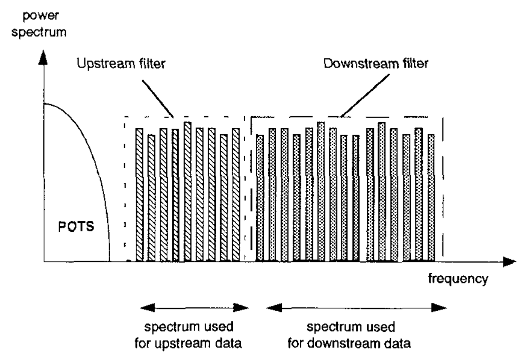

figure 2

In this paper, we want to illustrate the impact of the FDM-option on the equalization problem. In section 2, we start with an introduction of the reference equalization architecture before looking in section 3 at a reference algorithmic model and different implementations for retrieving the equalizer coefficients. In section 4, we demonstrate the insufficiency of the classical error-criterion in case of FDM. Section 5 proposes a new error-criterion that has been implemented in the Alcatel ADSL system.

0-7803-3250-4/96$5.00©1996 IEEE

2. DMT-ADSL EQUALIZATION PRINCIPLE

Discrete Multi-Tone (DMT) modulation is a common form of multi-carrier modulation. At the transmitter, the incoming data bits are split up in groups and mapped to successive DMT symbols. The data bits of one DMT symbol are modulated as 2^N-QAM on the different carriers by an Inverse Fast Fourier Transform (IFFT). At the receiver the bits are retrieved by the inverse operation, the Fast Fourier Transform (FFT) (figure 3).

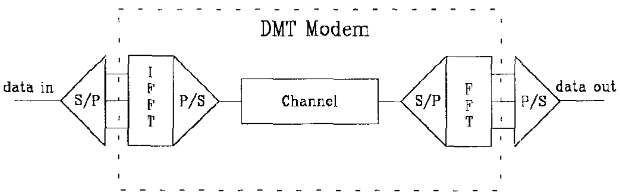

figure 3

Due to the frequency dependent channel attenuation and transmission delay, the transmitted signals are distorted and spread out, causing the end of one symbol to overlap with the beginning of the subsequent symbol (figure 4).

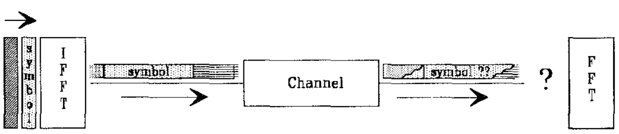

figure 4

To avoid Inter-Symbol-Interference (ISI) and Inter-Carrier-Interference (ICI) at the output of the FFT, a guardtime or guardband is inserted at the transmitter between two successive symbols. The purpose of the guardband is to absorb the ISI. This guardband is stripped off before the demodulating FFT. It consists of a short DMT symbol preamble that contains a replica of the last samples of the DMT symbol (figure 6).

Therefore each incoming DMT symbol can be treated as if it were a symbol extracted from a periodic DMT symbol stream and one can benefit from the convolution characteristics of the (I)FFT [3].

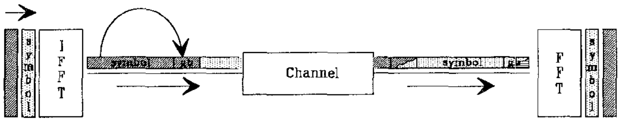

figure 5

The major draw-back of this simple ISI-cancelling technique is that it reduces the overall data throughput efficiency because a fraction of the transmission time is now used to transmit redundant information. The highly dispersive ADSL reference loops require a significant guardband-length which results in an intolerable reduction of the throughput efficiency. Increasing the DMT symbol-size in order to obtain an acceptable efficiency will mostly be inappropriate because of the increasing processing delay and hardware cost. A different approach proposed by Chow [4][5][6], consists of the use of a small Finite Impulse Response (FIR) filter, called Time domain EQualizer (TEQ). The objective of this simple tapped delay line is to compress the impulse response of the channel to a shorter overall impulse response. This compression results in a more time localized ISI and allows the use of a smaller guardband.

Once the ISI has been removed by extraction of the guardband, the carriers at the output of the FFT-block can be treated independently to compensate for the remaining frequency dependent attenuation and phase rotation of each carrier. This can be done by means of a one-tap complex filter bank called Frequency domain EQualizer (FEQ) (figure 6).

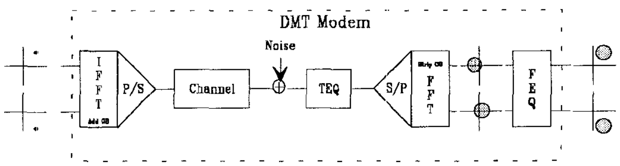

figure 6

3. TEQ-FEQ ALGORITHMIC MODEL

How does one find the optimal TEQ and FEQ coefficients under given hardware (complexity) constraints? It is obvious that once the TEQ has been derived, the FEQ can be retrieved easily. Indeed, for each tone the FEQ in figure 6 needs to compensate for the remaining rotation and amplitude deviation, which determines unambiguously the complex value of the corresponding tap. Notice that the FEQ affects both the signal and noise in the same proportion, therefore having no impact on the overall Signal to Noise Ratio (SNR) and thus on the capacity of the individual carriers.

In the literature various time- and frequency domain, adaptive, recursive or one-step approaches were considered. These approaches are basically inspired by the reference model used for Maximal Likelihood Sequence Detection Receivers [6][7][8]. The goal is to compress with a TEQ with impulse response w_k, the Channel Impulse Response (CIR) h_k to a Shortened Impulse Response (SIR). The SIR (result of the upper branch of figure 7) should match the best it can to a Target Impulse Response (TIR) b_k that fits within the guardband (lower branch of figure 7). Notice that the optimization of the TEQ, implies the optimization of the originally unknown TIR and of a delay parameter Δ.

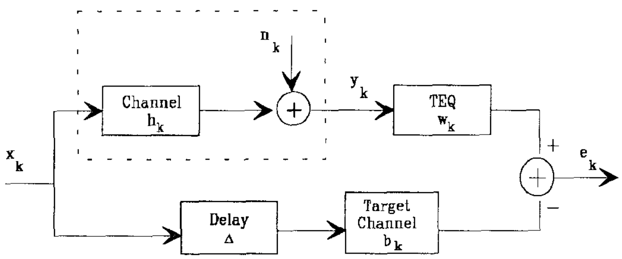

figure 7

The relationship between the transmitted sequence x_k, the additive channel noise n_k and the received sequence y_k can be formulated as

$$y_k = h_k * x_k + n_k \tag{1}$$

where $*$ denotes linear convolution.

To train the TEQ at start-up, ANSI specified the usage of a known pseudo-random training sequence with the periodicity of one DMT symbol. Notice that no guardband is used, nor needed, because of the periodicity of the training sequence.

Let's assume that the maximal CIR-length M is limited by the DMT-symbol size N and define a representative error [1] sequence e_k as

$$e_k \equiv w_k * y_k - b_k * x_{k-\Delta} = W^T.Y_k - B^T.X_k \tag{2}$$

[1] In most reference articles the optimization criterion reflects the amount of energy of the overall upper branch impulse response that falls outside the guardband, normed over the total energy of the upper branch impulse response. However, if the ISI can nearly be suppressed completely, both criteria give the same result.

with $W = \left[w_0 \;\; \dots \;\; w_{T-1}\right]^T$ where T equals the number of TEQ taps,

$B = \left[b_0 \;\; \dots \;\; b_{\nu-1}\right]^T$ where ν equals the TIR-or guardband length,

$Y_k = \left[y_k \;\; \dots \;\; y_{k-T+1}\right]^T$

and $X_k = \left[x_{k-\Delta} \;\; \dots \;\; x_{k-\Delta-\nu+1}\right]^T$

As optimization criterion, it was proposed to search for a TEQ, a SIR and a delay Δ that minimize the Mean Square Error (MSE) defined as

$$\mathrm{MSE} \equiv E\left\{e_k^2\right\} \overset{WSS}{=} E\left\{e^2\right\} = E\left\{\left(W^T.Y - B^T.X\right)^2\right\} \tag{3}$$

Defining $R_{YY} \equiv E\left\{Y.Y^T\right\}$, $R_{YX} \equiv E\left\{Y.X^T\right\}$ and

$$R_{XX} \equiv E\left\{X.X^T\right\} \tag{4}$$

the MSE can be rewritten as

$$\mathrm{MSE} = W^T.R_{YY}.W + B^T.R_{XX}.B - 2.W^T.R_{YX}.B \tag{5}$$

This MSE can be used to iteratively update W and B using e.g. a classical Least Mean Square (LMS) algorithm [9]. However, in order to avoid the slow convergence of the time domain-based iterative algorithms, an alternative, frequency domain approach was proposed in [4].

If the additive channel noise is neglected or averaged out, the incoming signal y_k at the input of the TEQ also exhibits symbol-periodicity when a (pseudo-random) training sequence x_k with a periodicity of one DMT symbol is sent out. The above error-equation can now be rewritten in terms of circular convolutions, denoted as $\otimes$, instead of linear convolutions :

$$e_k = w_k \otimes y_k - b_k \otimes x_{k-\Delta} \tag{6}$$

After FFT transformation (with size N) one obtains

$$E(i) = W_{win}(i).Y(i) - B_{win}^{\Delta}(i).X(i) \quad \forall i \in [0, N-1] \tag{7}$$

where i denotes the tone index. The subscript *win* refers to the fact that the sequences w_k and b_k are restricted to a time window of respectively maximal T and ν non-zeros values. To simplify the notation, the delay Δ has been included in a new shifted target transfer function. The new frequency domain optimization parameter is given by

$$\mathrm{MSE}(i) \equiv E\left\{\left|E(i)\right|^2\right\} = E\left\{\left|W_{win}(i).Y(i) - B_{win}^{\Delta}(i).X(i)\right|^2\right\}$$

$$\forall i \in [0, N-1] \tag{8}$$

This frequency domain MSE can be used as an adaptation parameter for an iterative block-LMS [10] update of alternatively $W(i)$ and $B(i)$.

To avoid the trivial "anti"-solution of a full-zero TEQ (w_k or $W(i)$) which delivers a zero MSE, both time and frequency domain approaches norm to unity the TEQ or TIR. In this paper, we have chosen to search for an optimal unity norm TEQ and request that, in the time domain approach

$$W_{opt}^T . W_{opt} = 1 \tag{9}$$

Making abstraction of the applied search algorithm (time or frequency domain, one step or iterative), the optimal TEQ solution complies to the following three conditions :

$$\left.\frac{\partial(\mathrm{MSE})}{\partial(B)}\right|_{B_{opt}, W_{opt}} = 0 \tag{10}$$

$$\left.\frac{\partial(\mathrm{MSE})}{\partial(W)}\right|_{B_{opt}, W_{opt}} = 0 \tag{11}$$

and $W_{opt}^T . W_{opt} = 1$ (12)

The optimal unity-norm TEQ W_{opt} is the unity-norm eigenvector of the Performance Matrix O belonging to the lowest eigenvalue of O [8][9]:

$$(O - \lambda_{\min} . I).W_{opt} = 0 \quad \text{and} \quad W_{opt}^T . W_{opt} = 1 \tag{13}$$

with $O \equiv R_{YY} - R_{YX} . \left(R_{XX}{}^{-1}\right)^T . R_{YX}{}^T$ (14)

If one assumes Wide Sense Stationary (WSS) signals and the noise independent of the transmitted training sequence, R_{XX}, R_{YX} and R_{YY} can be rewritten as [11]

$$R_{XX} = \begin{bmatrix} R_{xx}(0) & \cdots & R_{xx}(\nu-1) \\ \vdots & \ddots & \vdots \\ R_{xx}(\nu-1) & \cdots & R_{xx}(0) \end{bmatrix} \tag{15}$$

with $R_{xx}(k) \equiv E\{x_{i+k} . x_i\}$

$$R_{YX} = \begin{bmatrix} R_{yx}(\Delta) & \cdots & R_{yx}(\Delta+\nu-1) \\ \vdots & \ddots & \vdots \\ R_{yx}(\Delta-T+1) & \cdots & R_{yx}(\Delta+\nu-T) \end{bmatrix} \tag{16}$$

with $R_{yx}(k) \equiv E\{y_{i+k} . x_i\} = R_{xx}(k) * h_k$

$$R_{YY} = \begin{bmatrix} R_{yy}(0) & \cdots & R_{yy}(T-1) \\ \vdots & \ddots & \vdots \\ R_{yy}(T-1) & \cdots & R_{yy}(0) \end{bmatrix}$$

with $R_{yy}(k) \equiv E\{y_{i+k} . y_i\} = R_{xx}(k) * r_h(k) + R_{nn}(k)$ (17)

and $$r_h(k) = h_k * h_{-k} = \sum_{i=0}^{M-1} h_i . h_{i+k} \tag{18}$$

$$R_{nn}(k) = E\{n_{i+k} . n_i\} \tag{19}$$

R_{YY} can be split in a noise-free receiver autocorrelation matrix $R_{YY}{}^{nf}$ and a noise autocorrelation matrix R_{NN}

$$R_{YY} = R_{YY}{}^{nf} + R_{NN} \tag{20}$$

whose elements are represented respectively by the first and second term on the right side of equation (17).

4. TEQ TRAINING FOR FDM-BASED DMT-ADSL

We can now apply the formerly known algorithm described above on a typical FDM-based DMT-ADSL downstream channel whose magnitude is shown in figure 8. The total number of tones is assumed to be 256. The tones allocated to the downstream communication extend in this example from tone 46 to tone 255.

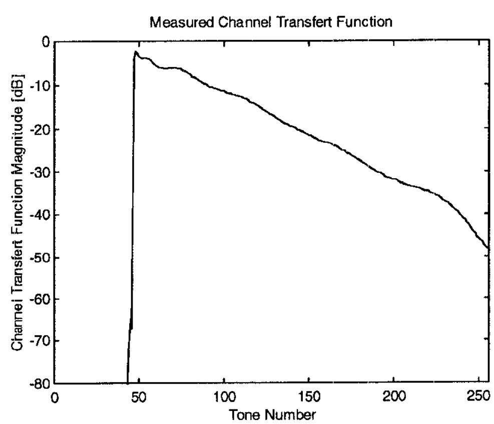

figure 8

For simplicity it is assumed that the channel is noise-free. The relevant normalized eigenvector of the related performance matrix is the optimal TEQ $\{w_k\}$. Its transfer function $W(i)$ is depicted in figure 9.

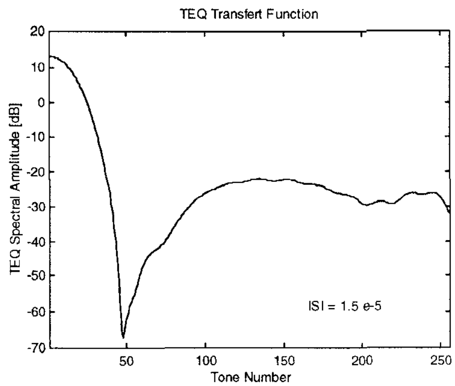

figure 9

Although this solution exhibits a low MSE, and a negligible ISI outside the guardband window, it results in a poor performance. The main cause is the relative concentration of the TEQ energy in the FDM-DMT stopband region (tones o to 45).

One might think that DMT modulation is not affected by the noise enhancement that typically occurs with a linear equalization. It appears as if the TEQ affects the energy of a narrowband carrier and the noise in the corresponding band in equal proportion, not deteriorating the original SNR of that carrier while removing the ISI. This is incorrect in reality [2]. At the receiver, the channel noise is spread by the FFT which acts like a filter bank. Each filter is centered at a carrier frequency and can be described by the following transfer function :

$$\frac{\sin(\pi.(f.T_c-k))}{\sin\left(\frac{\pi}{N}(f.T_c-k)\right)} \tag{21}$$

where k is the carrier number and $(T_c)^{-1}$ is the carrier spacing. Due to this spreading, commonly referred to as leakage, a narrowband noise hits multiple carriers. Consequently, the stopband noise (e.g. A/D noise) leaks into the passband (i.e. the downstream band in our example). In absence of a TEQ, this noise would be quite negligible. However with a TEQ that boosts the stopband noise by tens of dB's compared to the passband signal, this leakage causes the SNR of the used carriers to drop dramatically. The lowest downstream carriers are most severely damaged.

In ADSL, a pilot tone is used for clock synchronization. Mostly, this pilot is located in the lower region of the used band where the channel exhibits less attenuation. A SNR degradation on the pilot tone affects the synchronization performance of the receiver and results in increased clock jitter. The higher the frequency of a tone, the more severe its SNR degradation caused by clock jitter will be. As a consequence, the higher frequency tones which are less influenced by direct noise leakage, might be degraded indirectly by an increased clock jitter.

We conclude that the relative concentration of the TEQ energy in the FDM stopband region reduces the SNR in the FDM passband.

5. CONSTRAINED MSE

The concentration of the TEQ energy in the FDM stopband can best be explained using a frequency domain approach. Let us split the channel transfer function $H(i)$ in a stopband and a passband section as imposed by the FDM-filters. Applying a vector-notation one may write

$$H^B = \begin{bmatrix} H(0) \\ H(1) \\ \vdots \\ H(N-1) \end{bmatrix} = \begin{bmatrix} H^{stop} \\ \hline H^{pass} \end{bmatrix} \approx \begin{bmatrix} 0 \\ \hline H^{pass} \end{bmatrix} \tag{22}$$

Thriving towards a zero error ($E(i) \approx 0$) results in

$$W_{win}(i).Y(i) \approx B^{\Delta}_{win}(i).X(i) \quad \forall i \in [0, N-1] \tag{23}$$

or equivalently

$$W_{win}(i).H(i) \approx B^{\Delta}_{win}(i) \quad \forall i \in [0, N-1] \tag{24}$$

which becomes in a vector-notation [3]

$$\begin{bmatrix} W^{stop}_{win} \\ \hline W^{pass}_{win} \end{bmatrix} . \begin{bmatrix} 0 \\ \hline H^{pass} \end{bmatrix} \approx \begin{bmatrix} B^{stop}_{win} \\ \hline B^{pass}_{win} \end{bmatrix} \tag{25}$$

This results in three conditions for $W(i)$:

$$W^{pass}_{win}.H^{pass} \approx B^{pass}_{win} \tag{26}$$

[2] It is correct for an infinite length FFT and thus an infinite length DMT symbol.

[3] The delay Δ has been omitted to simplify notations.

W_{win}^{stop} is unconstrained (note that $B_{win}^{stop} \approx 0$) (27)

$$\sum_{i=0}^{N-1} \left| W_{win}(i) \right|^2 = 1 \quad \text{(unit-energy constraint)} \tag{28}$$

One notes that, although the unity-norm condition avoids convergence towards the full-zero "anti"-solution, still it cannot avoid a near zero-TEQ in the useful frequency band and as a consequence a near-zero TIR ($B_{win}(i) \approx 0 \quad \forall i$). Therefore new TEQ conditions are required in order to concentrate the TEQ energy in the FDM passband region.

To solve this problem, the original MSE-criterion in the time domain is modified by adding an additional degree of freedom that will allow to shape the TEQ spectral response. The new Constrained MSE (CMSE) criterion in the time domain is given by

$$\mathrm{CMSE} = \mathrm{MSE} + \mu . E_{SR} \tag{29}$$

with μ = Suppression Factor

$$E_{SR} = \sum_{i \in SR} \left| W_{win}(i) \right|^2 \quad \text{where}$$

$$W_{win}(i) = \frac{1}{\sqrt{N}} \sum_{k=0}^{T-1} w_k . e^{-j.2.\pi.\frac{k.i}{N}} \tag{30}$$

E_{SR} refers to the energy of the TEQ in the so-called Suppression Region (SR). The Suppression Factor μ determines how much the energy in SR has to be suppressed. Using matrix notation, one obtains

$$E_{SR} = W^T . \left(F_{SR}^T \right)^* . F_{SR} . W = W^T . D . W \tag{31}$$

$$\text{with} \quad F_{SR} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & e^{-j.2.\pi.\frac{a}{N}} & \cdots & e^{-j.2.\pi.\frac{a.(T-1)}{N}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j.2.\pi.\frac{b}{N}} & \cdots & e^{-j.2.\pi.\frac{b.(T-1)}{N}} \end{bmatrix} \tag{32}$$

where $[a, b] = SR$.

After substitution of (31) in (29) and making use of (5) we obtain

$$\mathrm{CMSE} = W^T . \left(R_{YY} + \mu . D \right) . W + B^T . R_{XX} . B - 2. W^T . R_{YX} . B \tag{33}$$

Notice that the matrix R_{YY} has been replaced by $R_{YY} + \mu . D$. Consequently, the eigenvector description representing the optimal TEQ is identical to the original one except for a modified performance matrix *O*:

$$\left(O - \lambda_{\min} . I \right) W_{opt} = 0 \quad \text{with}$$

$$O = \left(R_{YY} + \mu . D \right) - R_{YX} . R_{XX}^{-1} . R_{YX}^T \tag{34}$$

The additional term $\mu . D$ is identical to the insertion of a constant virtual noise power spectral density in the SR-frequency band (cf. figure 10):

$$\mu . D = R_{NN}{}^{virtual} = \begin{bmatrix} R_{nn}{}^{virtual}(0) & \cdots & R_{nn}{}^{virtual}(T-1) \\ \vdots & \ddots & \vdots \\ R_{nn}{}^{virtual}(T-1) & \cdots & R_{nn}{}^{virtual}(0) \end{bmatrix} \tag{35}$$

with $R_{nn}{}^{virtual}(k) = IFFT\left(PSD_{virtual\_noise}(i) \right)$ and

$$PSD_{virtual\_noise}(i) = \begin{cases} \mu & i \in SR \\ 0 & elsewhere \end{cases} \tag{36}$$

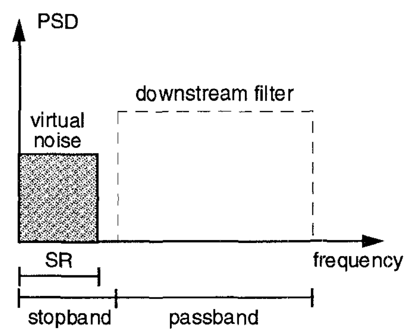

figure 10

The term '*virtual*' is used to indicate that it is not a real measured but a mathematically injected noise. As an illustration of the impact of the virtual noise injection artefact, figure 11 demonstrates the optimal TEQ with CMSE criterion. The SR band was selected from tone 0 up to tone 40. A bit-capacity increase per DMT symbol of approximately 35 % was realized on the reference loop depicted in figure 8 using this new CMSE error criterion.

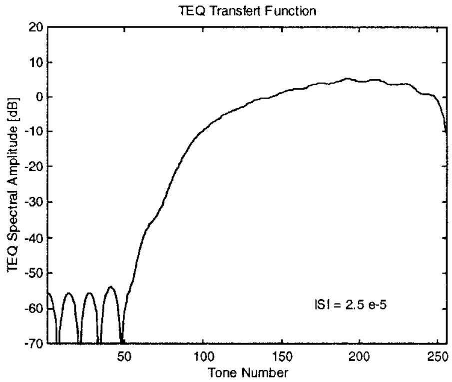

figure 11

Remark that the virtual noise injection technique can also be used with the frequency domain approach by modifying the frequency domain $\mathrm{MSE}(i)$ in a frequency domain $\mathrm{CMSE}(i)$:

$$\mathrm{CMSE}(i) = \mathrm{MSE}(i) + \mu(i).|W(i)|^2 \quad (37)$$

with $\mu(i) = \begin{cases} \mu & i \in SR \\ 0 & elsewhere \end{cases}$

6. FUTURE WORK

A more refined control of the virtual noise injection level depending on the line characteristics could be investigated. As an alternative one could use a shaped virtual noise PSD instead of a constant PSD level. The optimal injection level being the one that delivers the maximal overall bit capacity, which is the ultimate goal.

The latest TEQ equalization algorithm known to the author is given in [12] were the TEQ-optimality was also related to the maximal bit-allocation per DMT-symbol. In [12] however it is assumed that the entire bandwidth covered by the (I)FFT-modulation is used to carry the information bits. For FDM-based DMT-ADSL this prerequisite does not hold. Therefore, it could be interesting to relax some assumptions and study the impact of the unused frequency regions on the overall performance.

7. CONCLUSION

In this paper the unsatisfactory outcome of the existing equalization algorithms within the framework of FDM-based DMT-ADSL is demonstrated. Therefore a new Constrained MSE error-criterion has been proposed that is equivalent to the injection of virtual, mathematical noise in the unused spectral region. A computational efficient algorithm based on this technique was successfully implemented and resulted in a significant improvement of the overall bit-capacity.

ACKNOWLEDGEMENT

The authors would like to thank Prof. Marc Moeneclaey, Kristiaan Wuyts and Mark Van Bladel (University Ghent) for their valuable cooperation as well as Peter Reusens (Alcatel) for the many interesting discussions.

REFERENCES

[1] ANSI T1.413-1995, Asymmetric Digital Subscriber Line (ADSL) Metallic Interface.

[2] J. A. C. Bingham, "Multicarrier Modulation for Data Transmission: An Idea whose Time has Come", IEEE Communications Magazine, Vol. 28, p. 5-14, May 1990.

[3] E. A. Lee and D. G. Messerschmitt, "Digital Communication", Kluwer Academic Publisher, Boston, p. 255-260, p. 544-545, 1994.

[4] J. S. Chow, J. M. Cioffi and J. A. C. Bingham, "Equalizer Training Algorithms for Multicarrier Modulation Systems", Proc. IEEE, p. 761-765, 1993.

[5] J. S. Chow and J. M. Cioffi, "A Cost-Effective Maximum Likelihood Receiver for Multicarrier Systems", ICC 1992, p. 948-952, June 1992.

[6] J. S. Chow, J. C. Tu and J. M. Cioffi, "A Discrete Multitone Transceiver System for HDSL Applications", Journal on Selected Areas in Communications, Vol. 9, p. 895-908, August 1991.

[7] D. G. Messerschmitt, "Design of a Finite Impulse Response for the Viterbi Algorithm and Decision Feedback Equalizer", ICC 1974, p. 37D.1-37D.5, June 1974.

[8] D. D. Falconer and Jr. F. R. Magee, "Adaptive Channel Memory Truncation for Maximum Likelihood Sequence Estimation", Bell Syst. Tech. J. , p. 1541-1562, November 1973.

[9] B. Widrow and S. D. Stearns, "Adaptive Signal Processing", Prentice Hall, 1985.

[10] S. K. Mitra and J. F. Kaiser, "Handbook for Digital Signal Processing", 1993.

[11] W. A. Gardner, "Introduction to Random Processes", Mc. Graw-Hill, p. 189-195, 1989.

[12] N. Al-Dhahir and J. Cioffi, "Optimum Finite-Length Equalization for Multicarrier Transceivers", IEEE Globecom '94, p. 1884-1888.